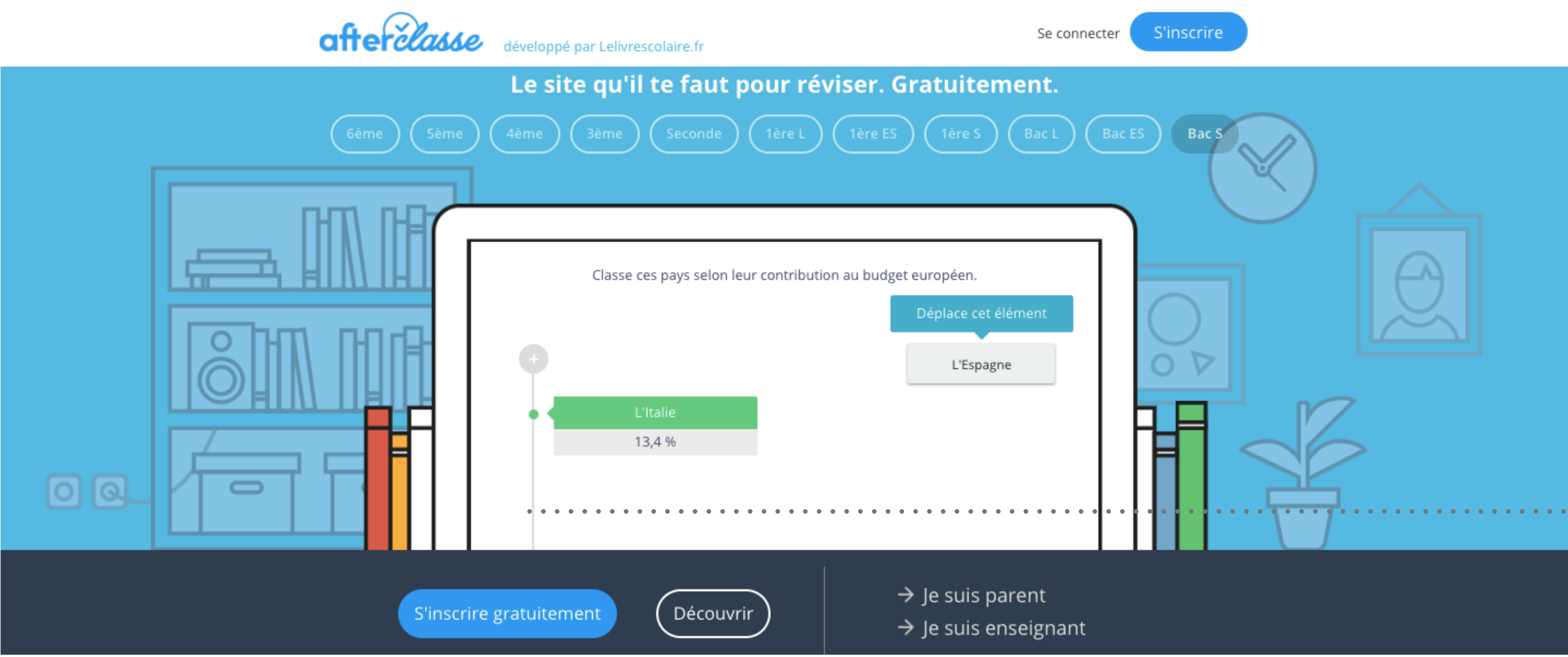


HOW DIFFICULT ARE ROTTING BANDITS?

Inria inventors for the digital world

https://www.afterclasse.fr

lelivrescolaire.fr

Julien SEZNEC

with A. Locatelli, A. Carpentier, A. Lazaric, M. Valko

SequeL @ Inria Lille — Nord Europe

WHEN BANDITS GO ROTTING …

WHEN BANDITS GO ROTTING …

2 days before the national exam

WHEN BANDITS GO ROTTING …

CHAPITRE 1

L'origine des séismes et des éruptions volcaniques

CHAPITRE 2

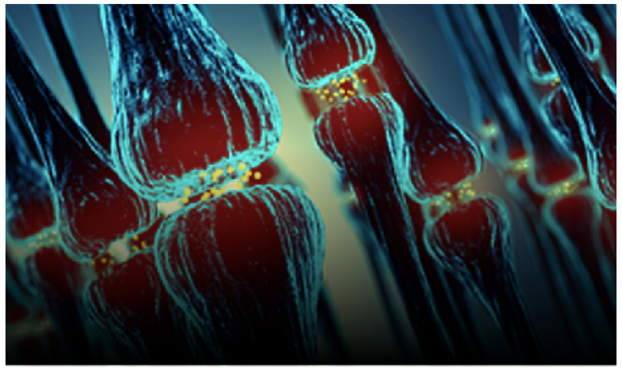
Les changements climatiques actuels et leurs conséquences

afterclasse

2 days before the national exam

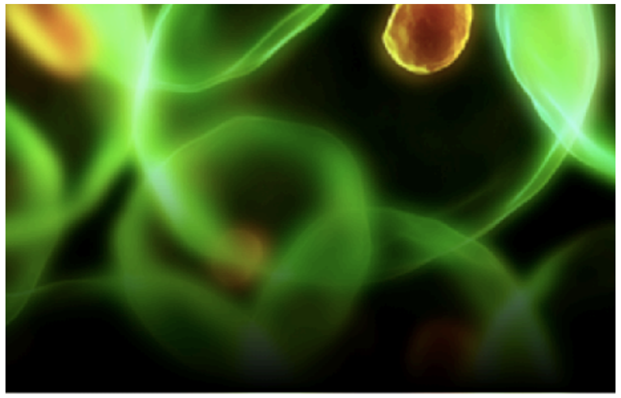
CHAPITRE 3

Les impacts des activités humaines sur l'environnement



CHAPITRE 8

Le fonctionnement du système nerveux



CHAPITRE 4

La nutrition à l'échelle cellulaire

Inria informatics mathematics

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ROTTING BANDITS ARE …

Stochastic bandits …

- K arms
- At each round t, agent pulls arm i and receives a noisy reward $r_t \leftarrow \mu_i + \epsilon_t$ (ϵ_t i.i.d. ;σ-subgaussian)
- Maximize cumulative reward : $\mathbb{E}\left[\sum_{t \leq T} r_t\right]$

ROTTING BANDITS ARE …

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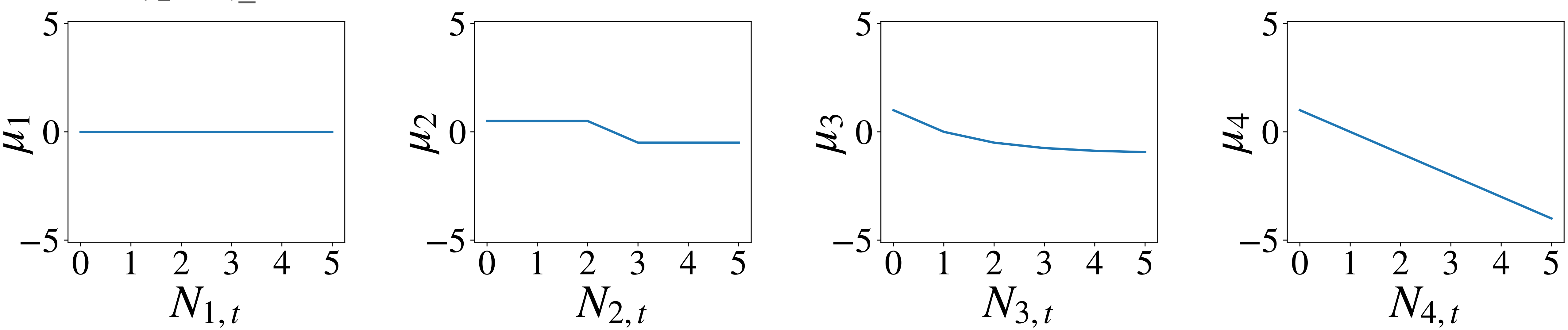
- Maximize cumulative reward : $\mathbb{E}\left[\sum_{t \leq T} r_t\right]$

… with rotting arms

- $\{\mu_i\}$ are ***non-increasing*** functions of $N_{i,t}$ the ***number of pulls of arm*** i at time t
- $L \triangleq \max_{i \in K} \max_{n \leq T} \mu_i(n) - \mu_i(n+1)$



OPTIMAL ORACLE POLICY [HEIDARI, 2016]

Algorithm 1 $\mathcal{A}_0$ (Heidari et al., 2016)

```
for t ← 1, 2, ... do
    SELECT : arg max_{i∈K} μ_i(N_{i,t})
end for
```

$$\textbf{for } t \leftarrow 1, 2, \ldots \textbf{ do} \quad \text{SELECT} : \arg\max_{i \in \mathcal{K}} \mu_i(N_{i,t}) \quad \textbf{end for}$$

OPTIMAL ORACLE POLICY [HEIDARI, 2016]

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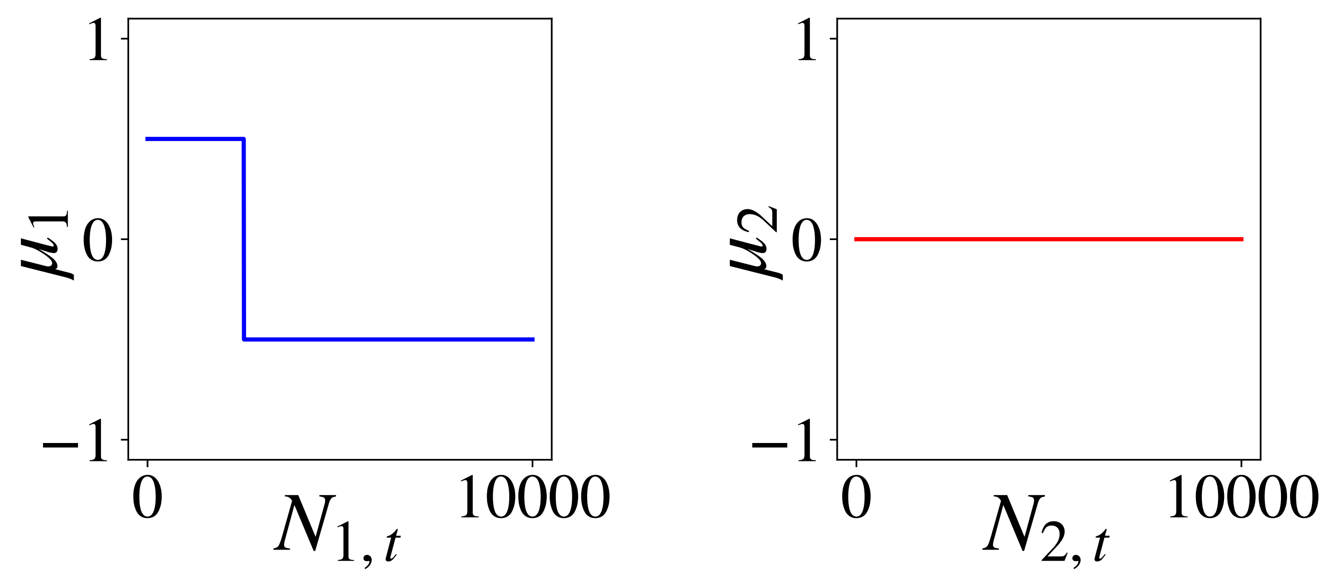

informatics mathematics

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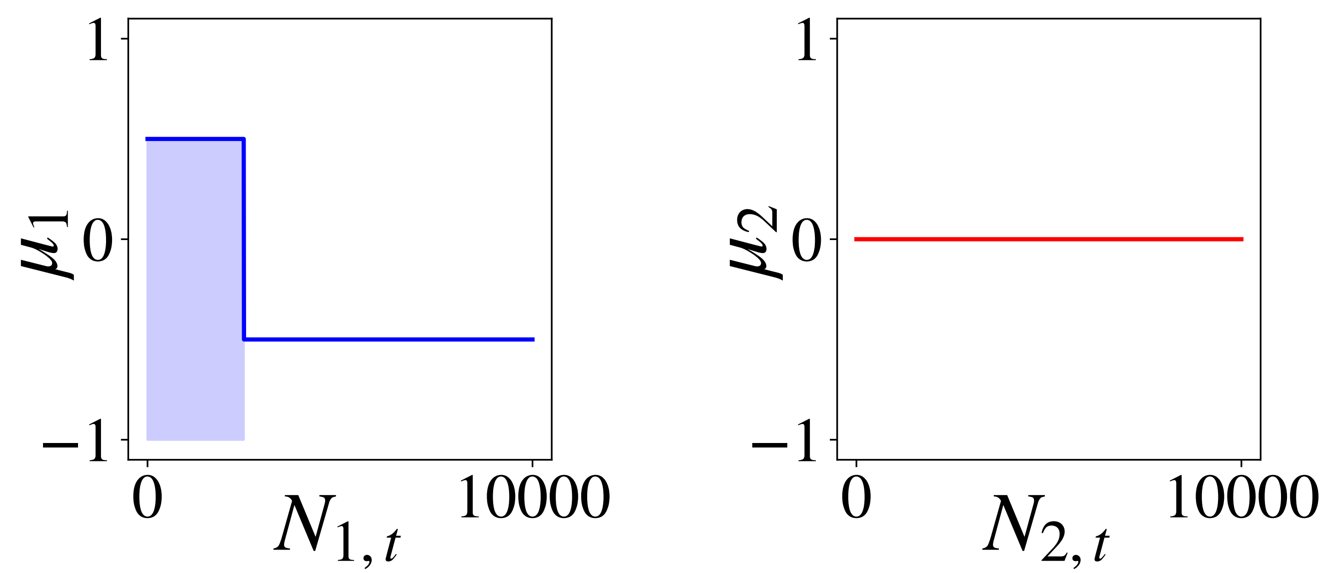

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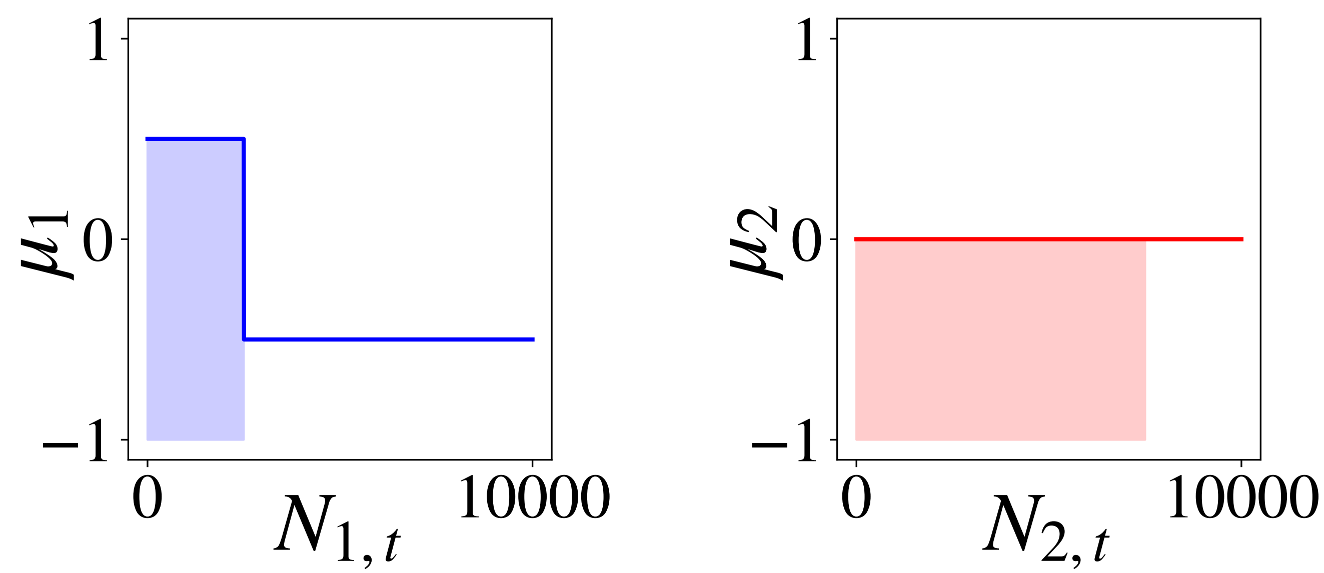

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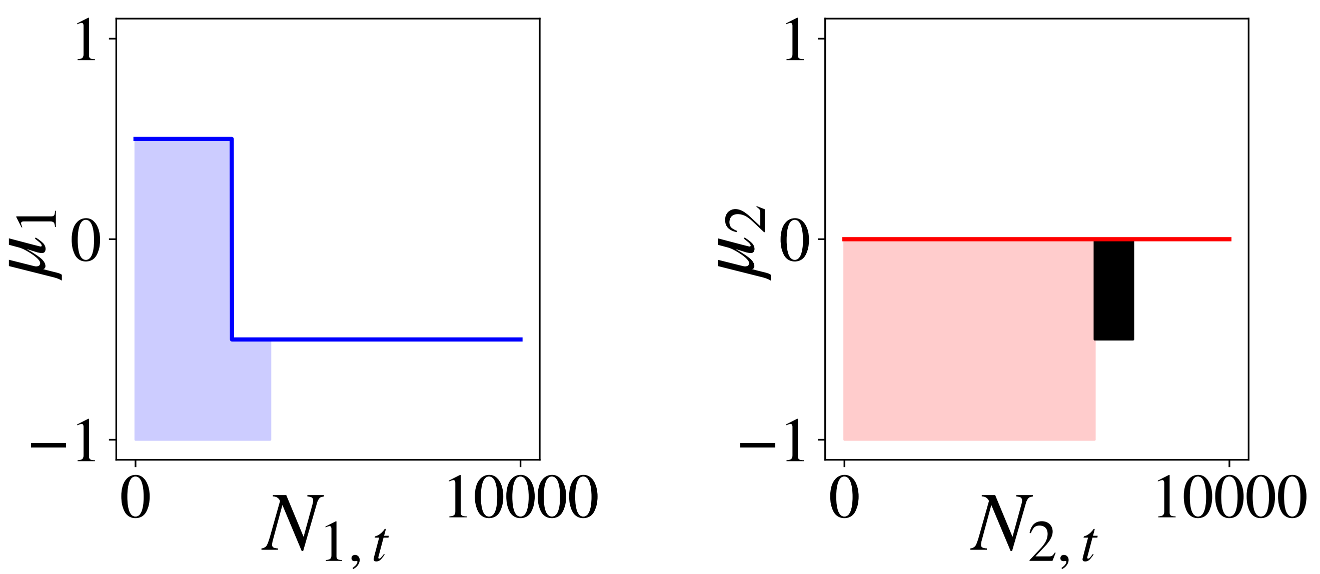

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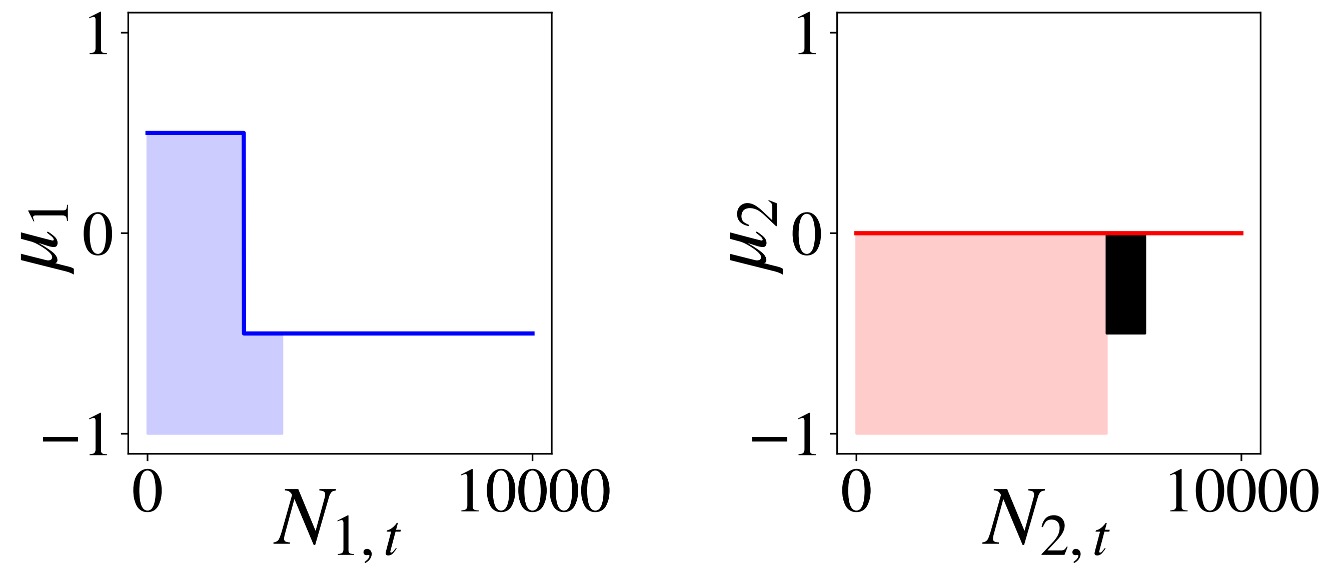
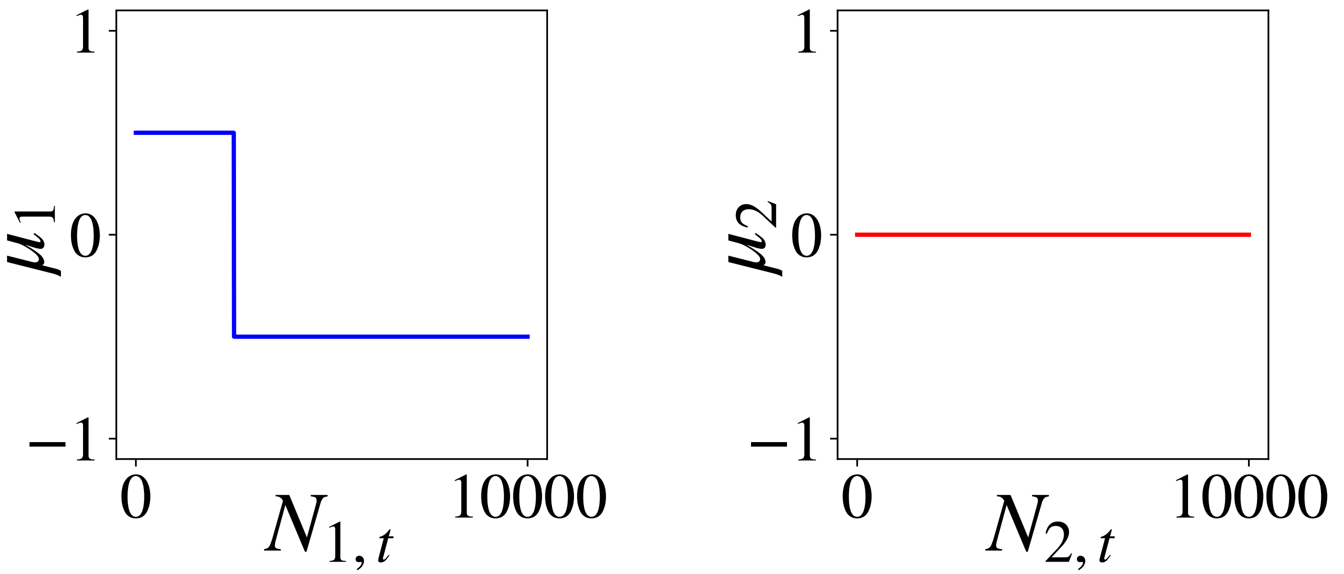


$$R_T(\pi) = \sum_{i \in \mathrm{UP}} \sum_{s=N^{\pi}_{i,T}+1}^{N^{\star}_{i,T}} \mu_i(s) - \sum_{i \in \mathrm{OP}} \sum_{s=N^{\star}_{i,T}+1}^{N^{\pi}_{i,T}} \mu_i(s)$$

NOISE-FREE BANDIT POLICY [HEIDARI, 2016]

Algorithm 2 $\mathcal{A}_2$ (Heidari et al., 2016)

1: **for** $t \leftarrow K+1, K+2, \ldots$ **do**
2: $\quad$ SELECT : $\arg\max_{i \in \mathcal{K}} \mu_i(N_{i,t} - 1)$
3: **end for**



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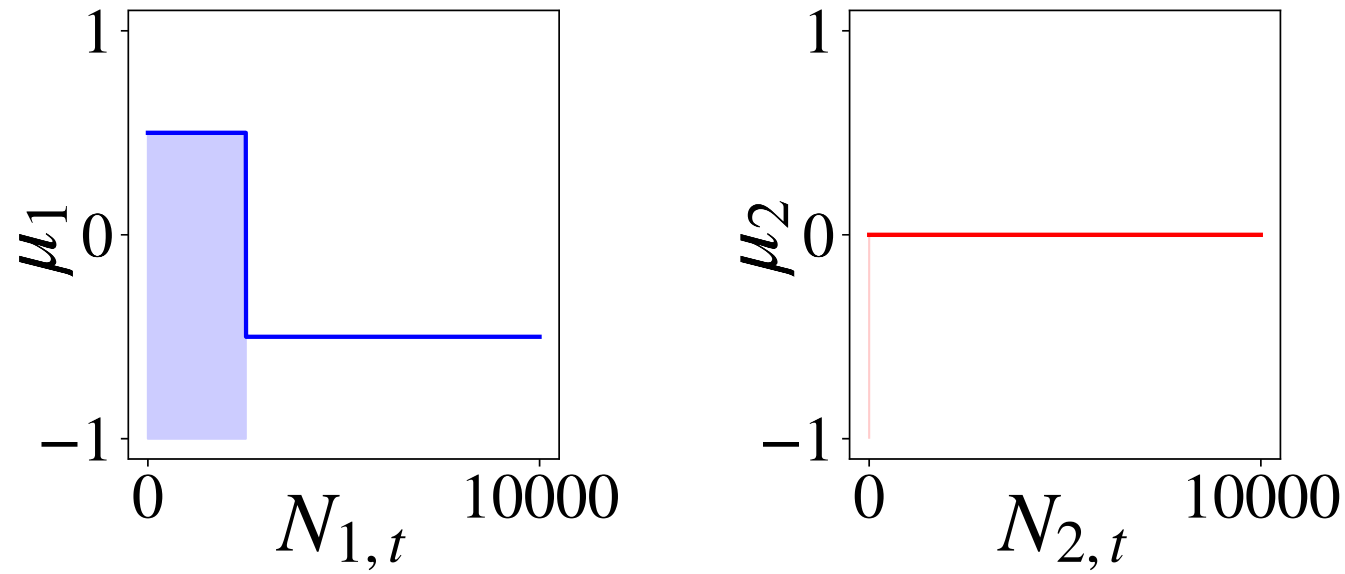

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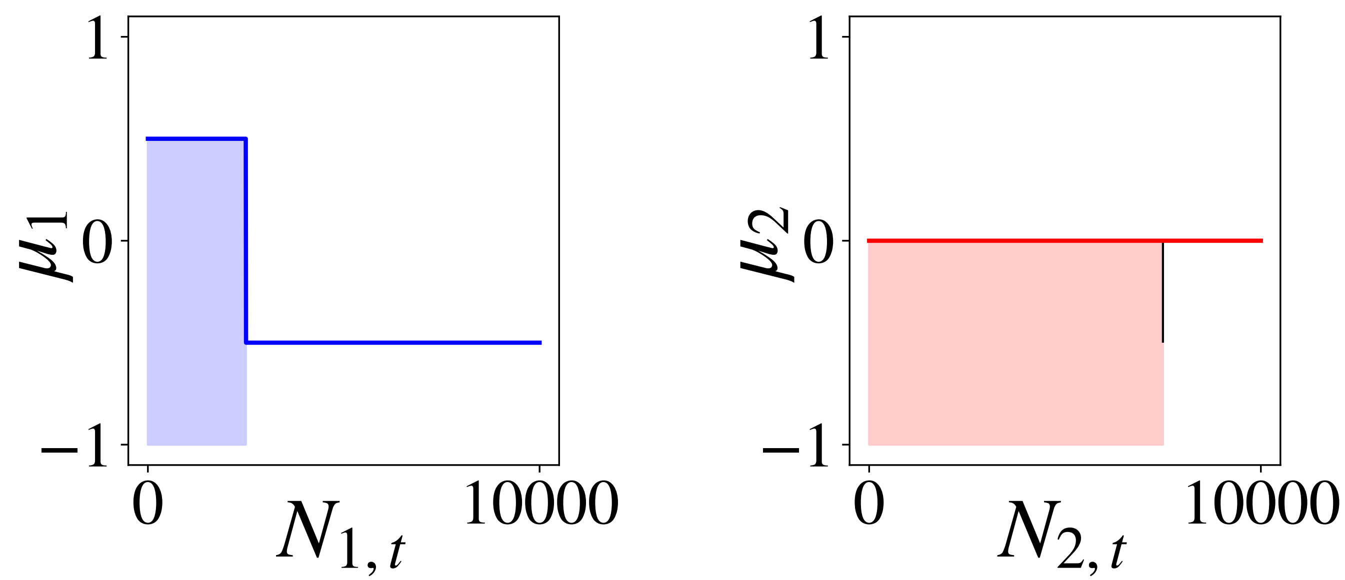

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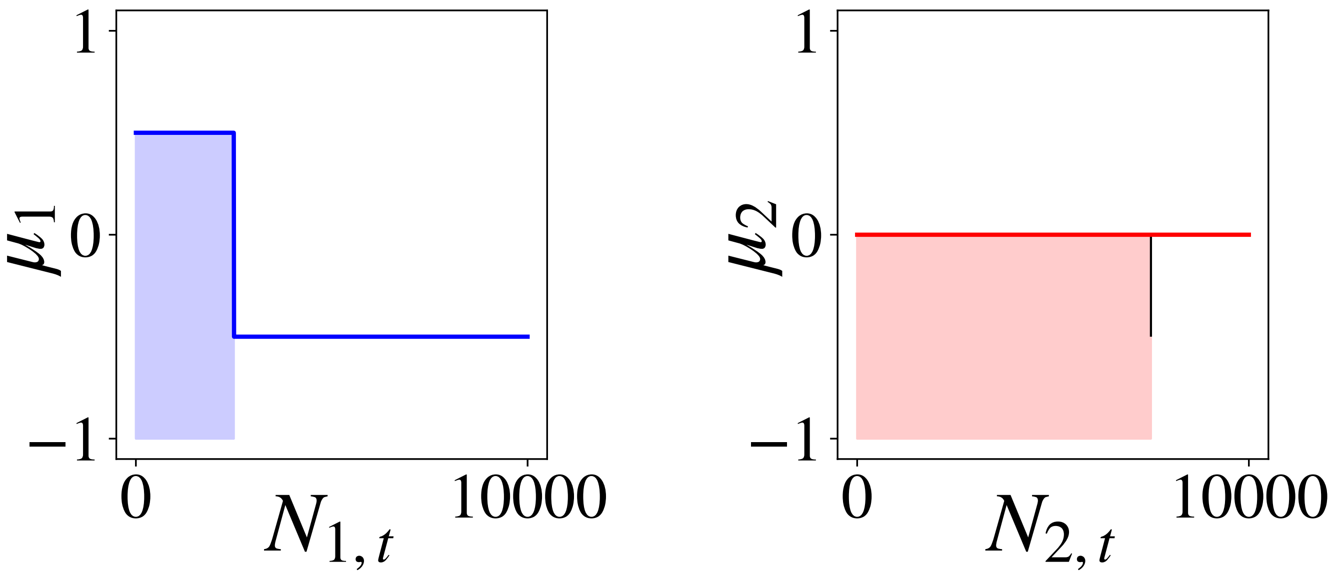


Worst-case minimax optimal rate : $R_T(\pi_{A_2}) \leq KL$

wSWA [LEVINE, 2017]

Algorithm 3 SWA (Levine et al., 2017)

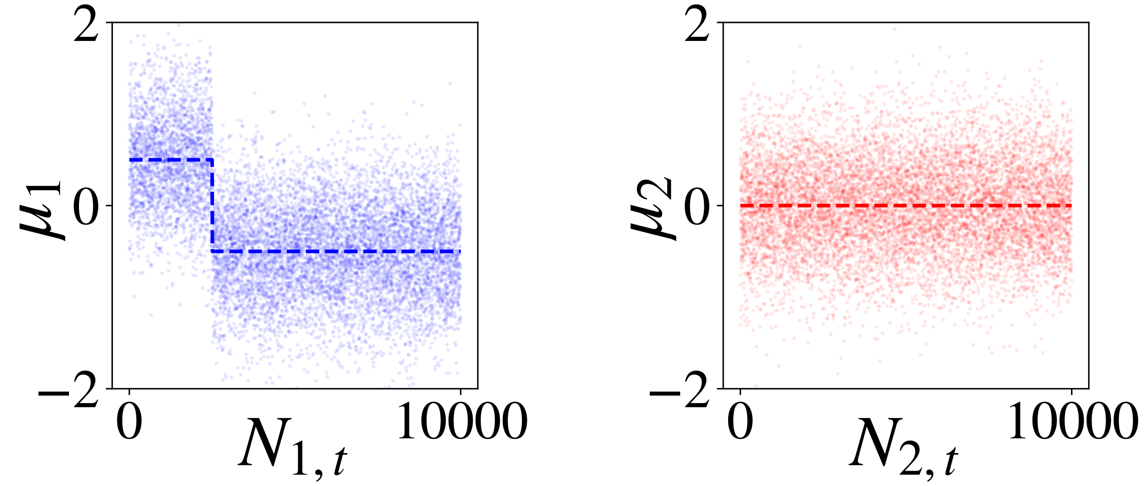
Input: K, L, T, σ

$h \leftarrow \widetilde{\mathcal{O}}\left(\left(\frac{\sigma T}{KL}\right)^{2/3}\right)$

for $t \leftarrow Kh+1, Kh+2, \ldots$ **do**

  SELECT : $\arg\max_{i \in \mathcal{K}} \widehat{\mu}_i^h(N_{i,t})$

end for



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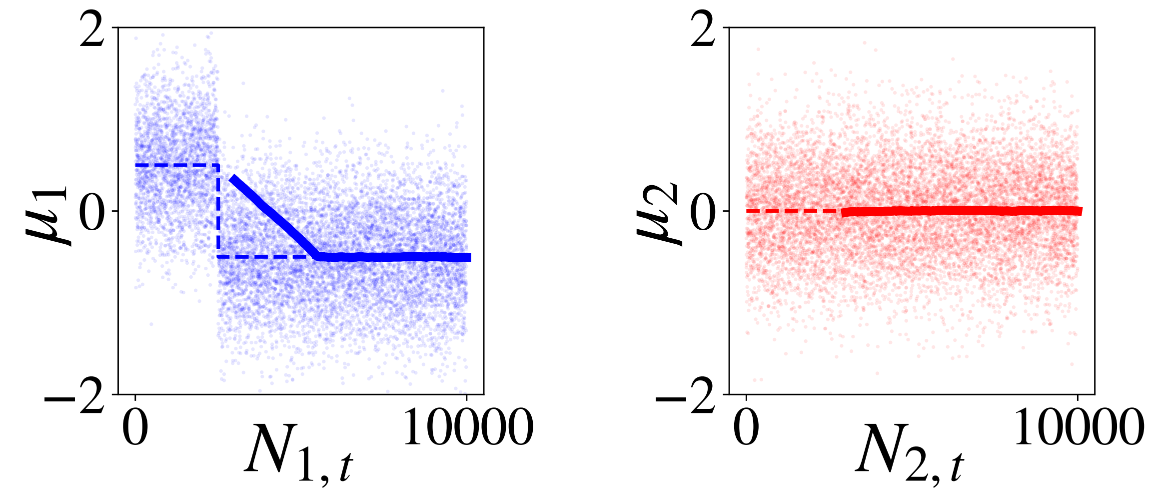


h = 3000

Regret due to bias: $\widetilde{O}\left(LKh\right)$

wSWA [LEVINE, 2017]

Algorithm 3 SWA (Levine et al., 2017)

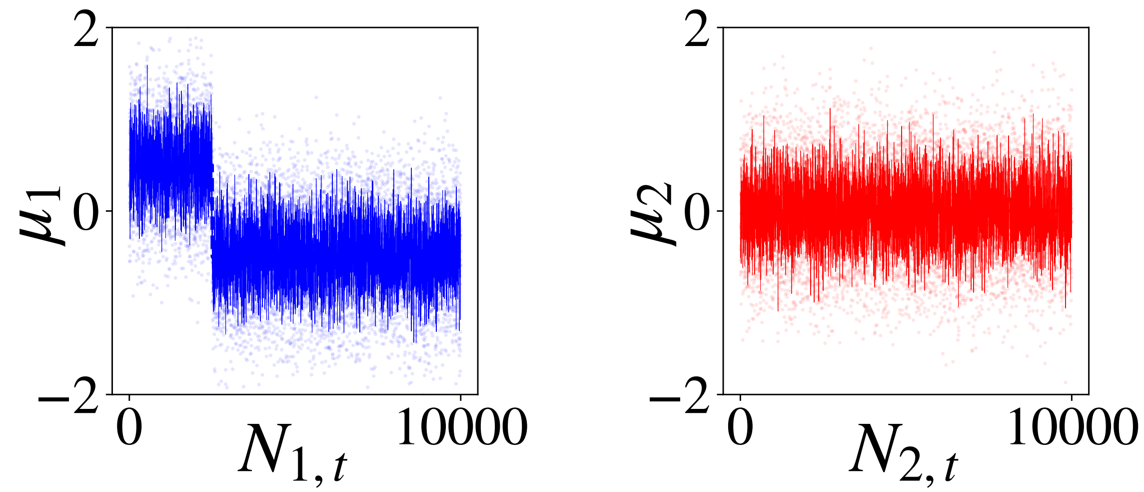
Input: K, L, T, σ

1: $h \leftarrow \widetilde{\mathcal{O}}\left(\left(\frac{\sigma T}{KL}\right)^{2/3}\right)$

2: **for** $t \leftarrow Kh+1, Kh+2, \ldots$ **do**

3: SELECT : $\arg\max_{i \in \mathcal{K}} \widehat{\mu}_i^h(N_{i,t})$

4: **end for**



h = 3

Regret due to bias: $\widetilde{O}\left(LKh\right)$

Regret due to variance : $\widetilde{O}\left(\sigma T\sqrt{h}^{-1}\right)$

wSWA [LEVINE, 2017]

Algorithm 3 SWA (Levine et al., 2017)

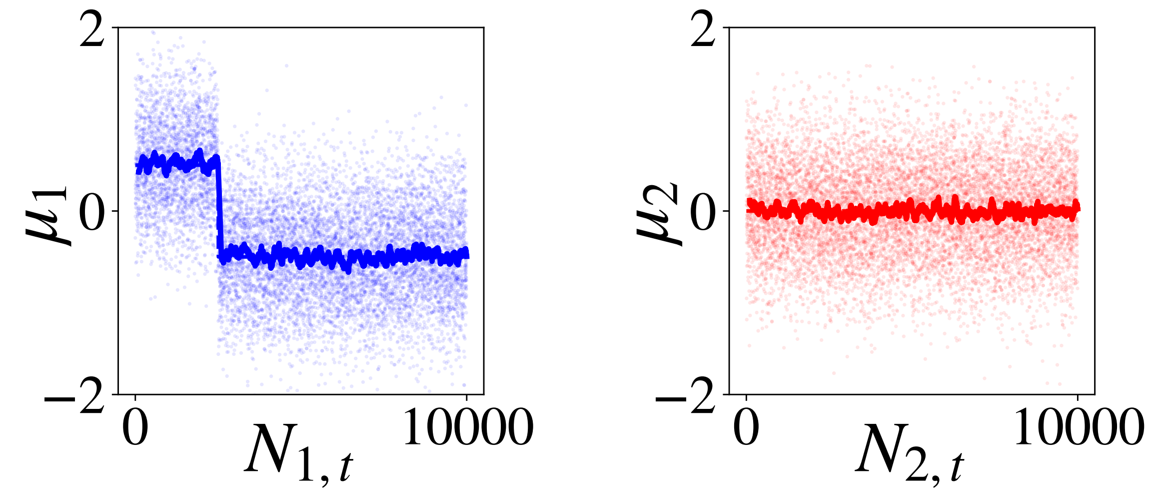
Input: K, L, T, σ

$h \leftarrow \widetilde{\mathcal{O}}((\frac{\sigma T}{KL})^{2/3})$

for $t \leftarrow Kh+1, Kh+2, \dots$ **do**

  SELECT : $\arg\max_{i \in \mathcal{K}} \widehat{\mu}_i^h(N_{i,t})$

end for



h = 100

Regret due to bias: $\widetilde{O}\left(LKh\right)$

Regret due to variance : $\widetilde{O}\left(\sigma T \sqrt{h}^{-1}\right)$

Worst case regret : $\widetilde{O}\left(K^{1/3}T^{2/3}\right)$

THE FAILURE OF wSWA

Sample	old																			last
Arm 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
Arm 2	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	1	1
Arm 3	**X**	**X**	1	1	1	1	1	1	1	1	0	1	1	0	1	1	0	1	1	0

THE FAILURE OF wSWA

Sample	old																			last
Arm 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
Arm 2	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	1	1
Arm 3	**X**	**X**	1	1	1	1	1	1	1	1	0	1	1	0	1	1	0	1	1	0

Won't we benefit from a data-adaptive window ?

FILTERING ON EXPANDING WINDOW AVERAGE (FEWA)

Algorithm 4 FEWA

Input: K, σ, α

for $t \leftarrow K+1, K+2, \dots$ **do**
  $\delta_t \leftarrow \frac{1}{Kt^\alpha}$
  $h \leftarrow 1$
  $\mathcal{K}_1 \leftarrow \mathcal{K}$
  do
    $\mathcal{K}_{h+1} \leftarrow \left\{ i \in \mathcal{K}_h \,|\, \widehat{\mu}_i^h(N_{i,t}) \geq \max_{j \in \mathcal{K}} \widehat{\mu}_j^h(N_{j,t}) - 2c(h, \delta_t) \right\}$
    $h \leftarrow h+1$
  while $h \leq \min_{i \in \mathcal{K}_h} N_{i,t}$
  SELECT : $\{ i \in \mathcal{K}_h \,|\, h > N_{i,t} \}$
end for

Sample	old																			last
Arm 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
Arm 2	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	1	1
Arm 3	X	X	1	1	1	1	1	1	1	1	0	1	1	0	1	1	0	1	1	0

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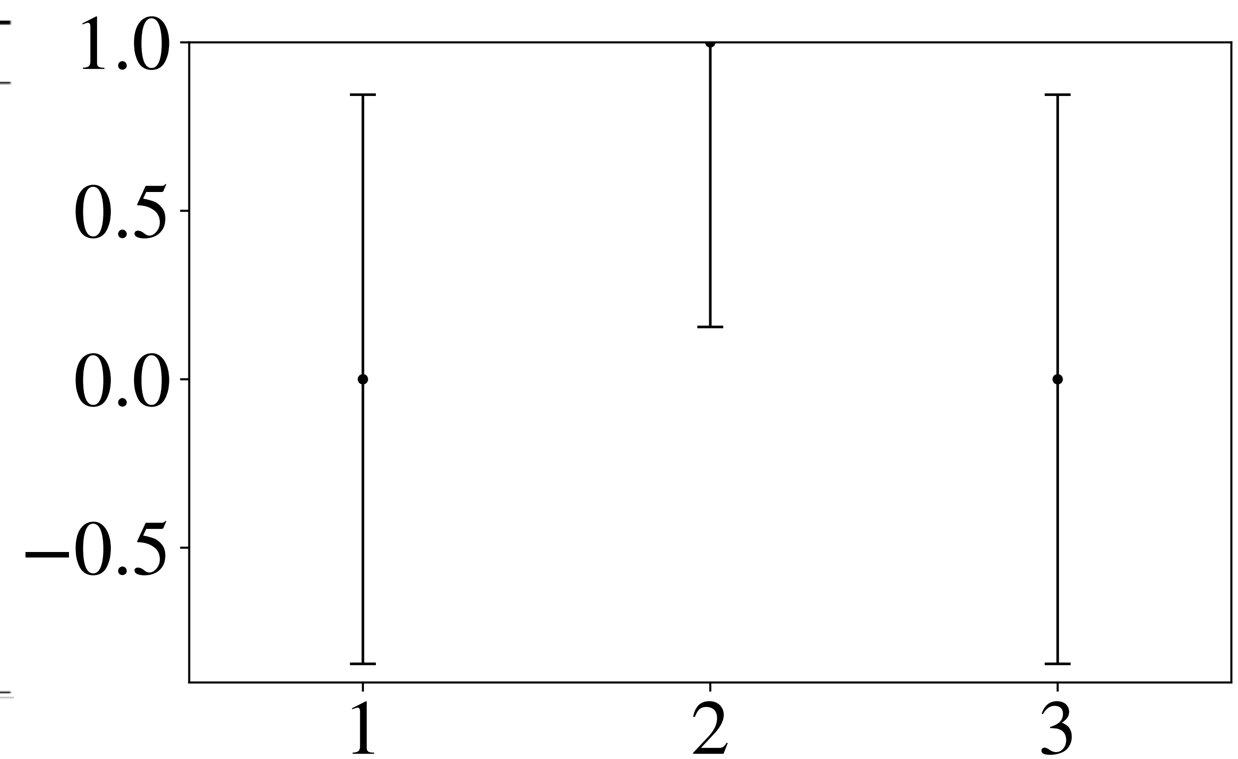
 while $h \leq \min_{i\in\mathcal{K}_h} N_{i,t}$
 SELECT : $\{i \in \mathcal{K}_h \,|\, h > N_{i,t}\}$
end for



Sample	old																			last
Arm 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
Arm 2	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	1	1
Arm 3	**X**	**X**	1	1	1	1	1	1	1	1	0	1	1	0	1	1	0	1	1	0

h = 1

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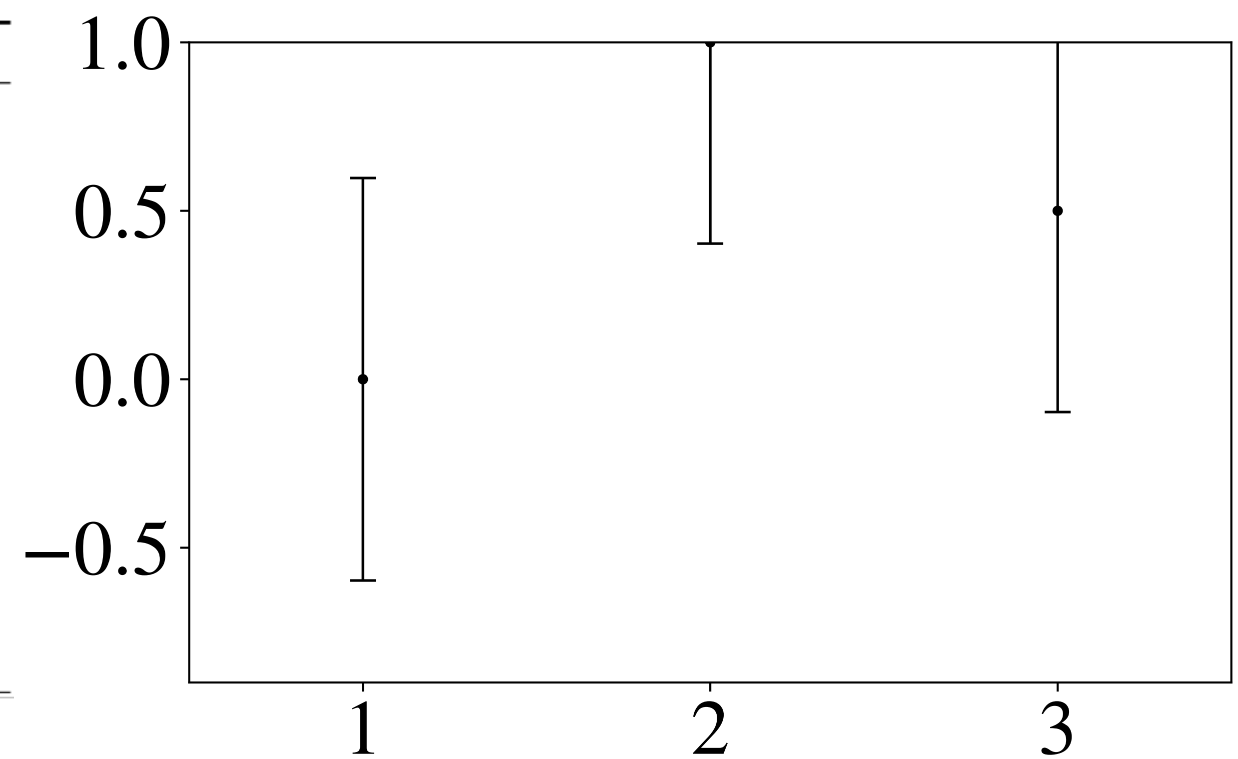
    $h \leftarrow h+1$

  while $h \leq \min_{i \in \mathcal{K}_h} N_{i,t}$

  SELECT : $\{i \in \mathcal{K}_h \,|\, h > N_{i,t}\}$

end for



Sample	old																		last
Arm 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
Arm 2	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	1	1
Arm 3	**X**	**X**	1	1	1	1	1	1	1	1	0	1	1	0	1	1	0	1	1	0

h = 2

FILTERING ON EXPANDING WINDOW AVERAGE (FEWA)

Algorithm 4 FEWA

Input: K, σ, α

for $t \leftarrow K+1, K+2, \dots$ **do**
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    $h \leftarrow h+1$
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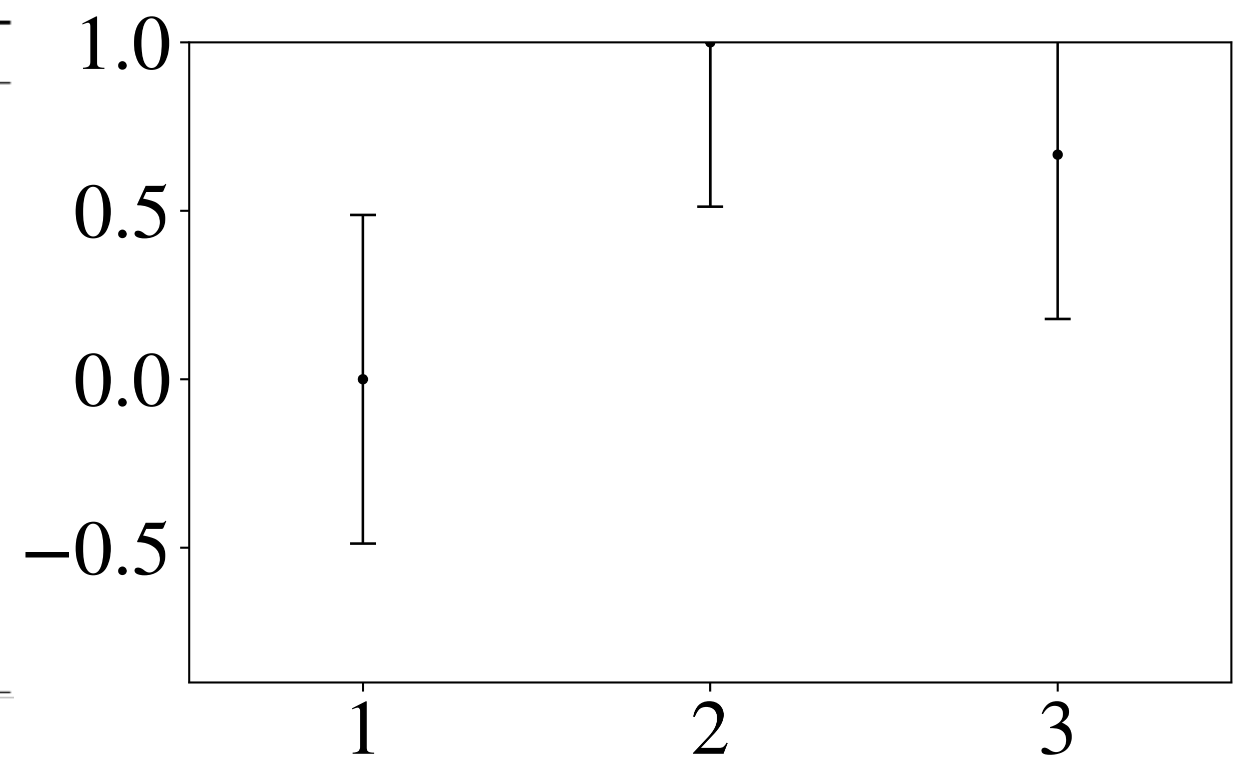
  SELECT : $\{i \in \mathcal{K}_h \,|\, h > N_{i,t}\}$
end for



Sample	old																			last
Arm 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
Arm 2	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	1	1
Arm 3	**X**	**X**	1	1	1	1	1	1	1	1	0	1	1	0	1	1	0	1	1	0

h = 3

FILTERING ON EXPANDING WINDOW AVERAGE (FEWA)

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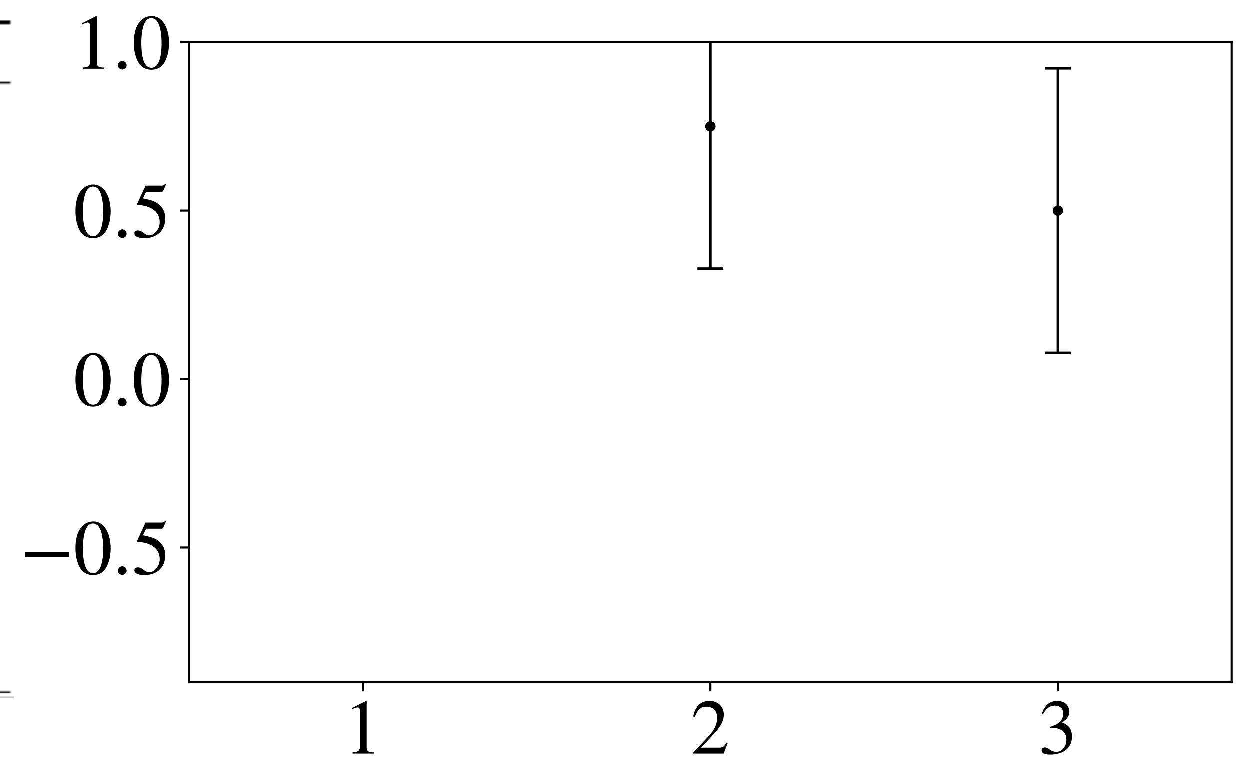
    SELECT : $\{ i \in \mathcal{K}_h \,|\, h > N_{i,t} \}$
end for



Sample	old																			last
Arm 1																				
Arm 2	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	1	1
Arm 3	**X**	**X**	1	1	1	1	1	1	1	1	0	1	1	0	1	1	0	1	1	0

h = 4

FILTERING ON EXPANDING WINDOW AVERAGE (FEWA)

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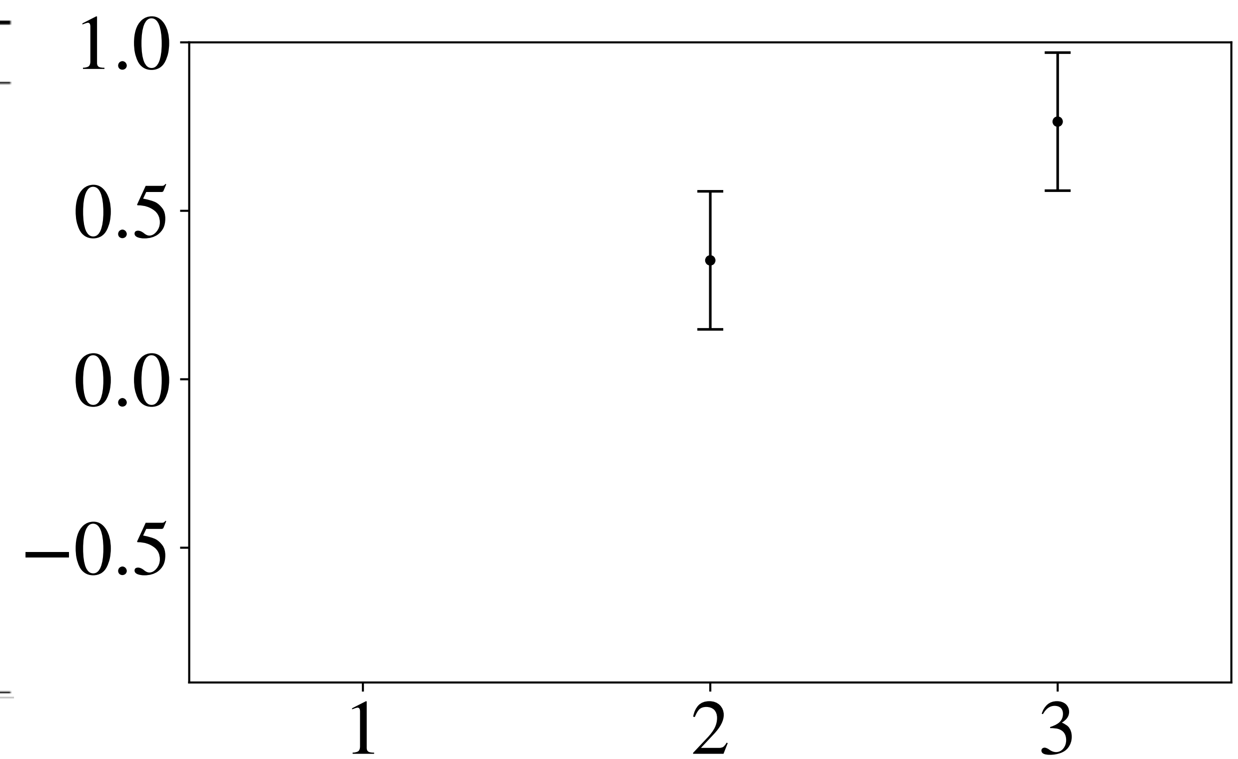
  SELECT : $\{i \in \mathcal{K}_h \,|\, h > N_{i,t}\}$
end for



Sample	old																			last
Arm 1																				
Arm 2	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	1	1
Arm 3	**X**	**X**	1	1	1	1	1	1	1	1	0	1	1	0	1	1	0	1	1	0

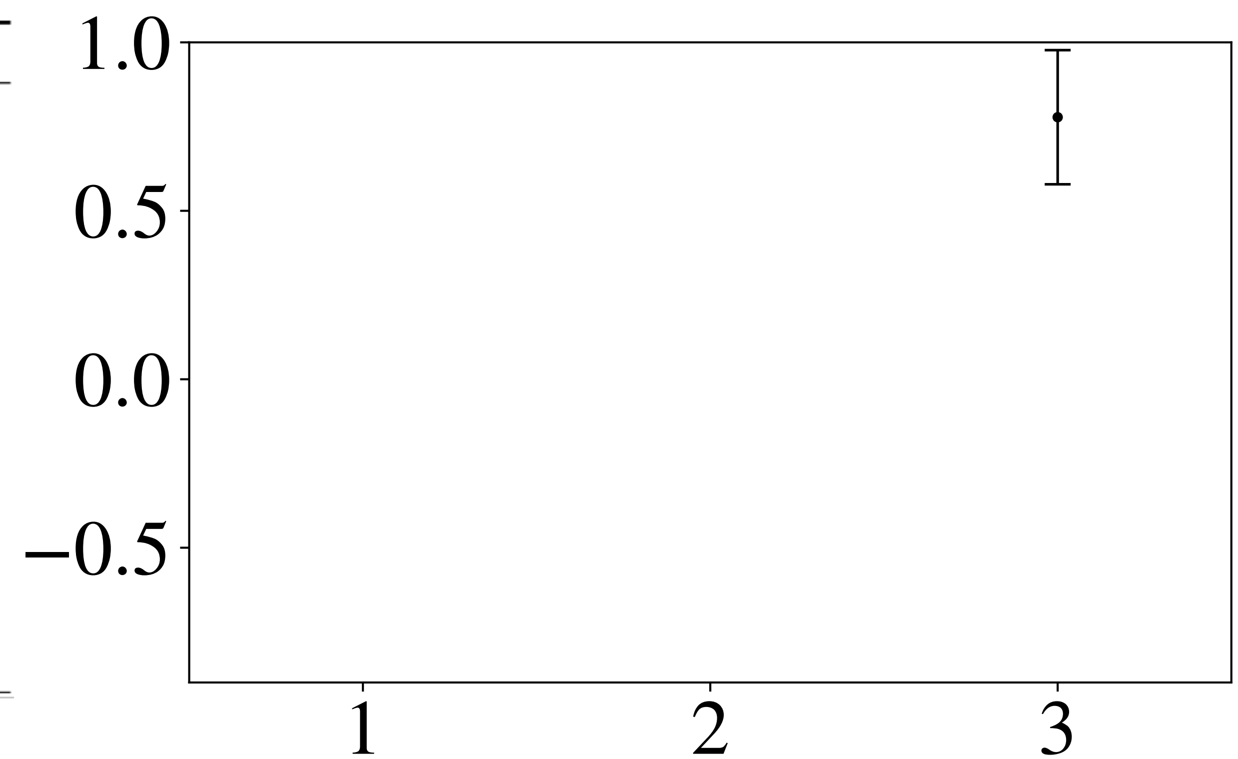
h = 17

FILTERING ON EXPANDING WINDOW AVERAGE (FEWA)

Algorithm 4 FEWA

Input: K, σ, α

for $t \leftarrow K+1, K+2, \dots$ **do**
  $\delta_t \leftarrow \frac{1}{Kt^\alpha}$
  $h \leftarrow 1$
  $\mathcal{K}_1 \leftarrow \mathcal{K}$
  do
    $\mathcal{K}_{h+1} \leftarrow \left\{ i \in \mathcal{K}_h \,|\, \widehat{\mu}_i^h(N_{i,t}) \geq \max_{j \in \mathcal{K}} \widehat{\mu}_j^h(N_{j,t}) - 2c(h, \delta_t) \right\}$
    $h \leftarrow h + 1$
  while $h \leq \min_{i \in \mathcal{K}_h} N_{i,t}$
  SELECT : $\{ i \in \mathcal{K}_h \,|\, h > N_{i,t} \}$
end for



Sample	old																			last
Arm 1																				
Arm 2																				
Arm 3	X	X	1	1	1	1	1	1	1	1	0	1	1	0	1	1	0	1	1	0

h = 18

UPPER BOUNDS

Worst-case upper bound

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{F}}\right)\right] \leq C\sigma\sqrt{KT\log(KT)} + KL$$

Comparison w/ wSWA

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{wSWA}}\right)\right] = \tilde{O}\left(L^{1/3}\sigma^{2/3}K^{1/3}T^{2/3}\right)$$

UPPER BOUNDS

Worst-case upper bound

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Comparison w/ wSWA

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{wSWA}}\right)\right] = \tilde{O}\left(L^{1/3}\sigma^{2/3}K^{1/3}T^{2/3}\right)$$

Problem-dependent upper bound

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{F}}\right)\right] \leq \sum_{i\in\mathcal{K}} O\left(\frac{\log(KT)}{\Delta_{i,h^+_{i,T}-1}}\right)$$

Comparison w/ wSWA

Pure worst-case strategy

$\Delta_{i,h}$ Difference between the average of the h first overpulls of arm i and the worst reward pulled by the optimal policy

$h^+_{i,T}$ High-probability upper bound on the number of overpulls for FEWA

UPPER BOUNDS

Worst-case upper bound

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{F}}\right)\right] \leq C\sigma\sqrt{KT\log(KT)} + KL$$

Comparison w/ wSWA

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{wSWA}}\right)\right] = \tilde{O}\left(L^{1/3}\sigma^{2/3}K^{1/3}T^{2/3}\right)$$

Problem-dependent upper bound

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{F}}\right)\right] \leq \sum_{i\in\mathcal{K}} O\left(\frac{\log(KT)}{\Delta_{i,h^+_{i,T}-1}}\right)$$

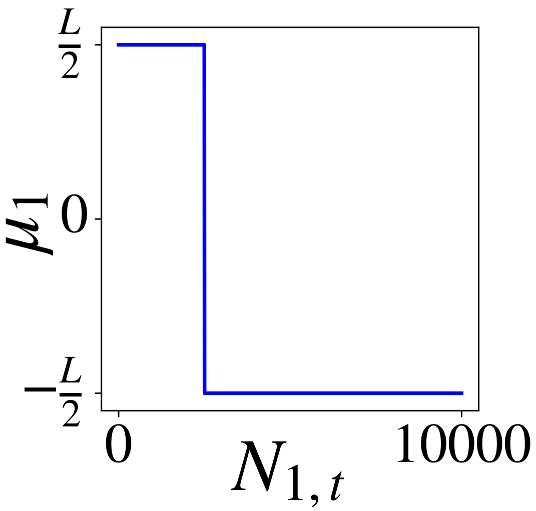
Comparison w/ wSWA

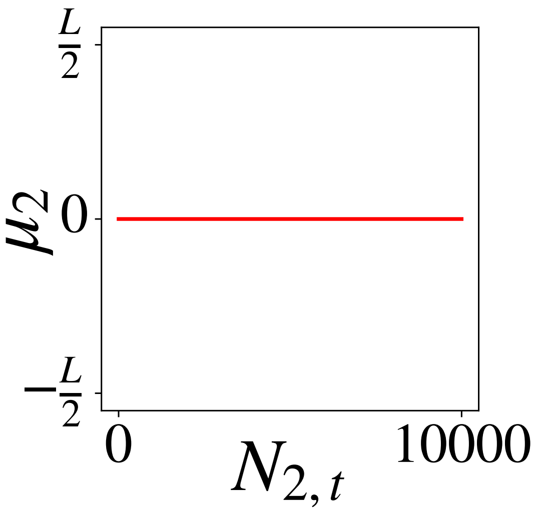
Pure worst-case strategy

$\Delta_{i,h} = \Delta_i$ on a stationner bandits problem

$\Delta_{i,h^+_{i,T}-1}$ is a problem-dependent quantity

2-ARMS EXPERIMENTS





$$\sigma = 1 \; ; \; L$$

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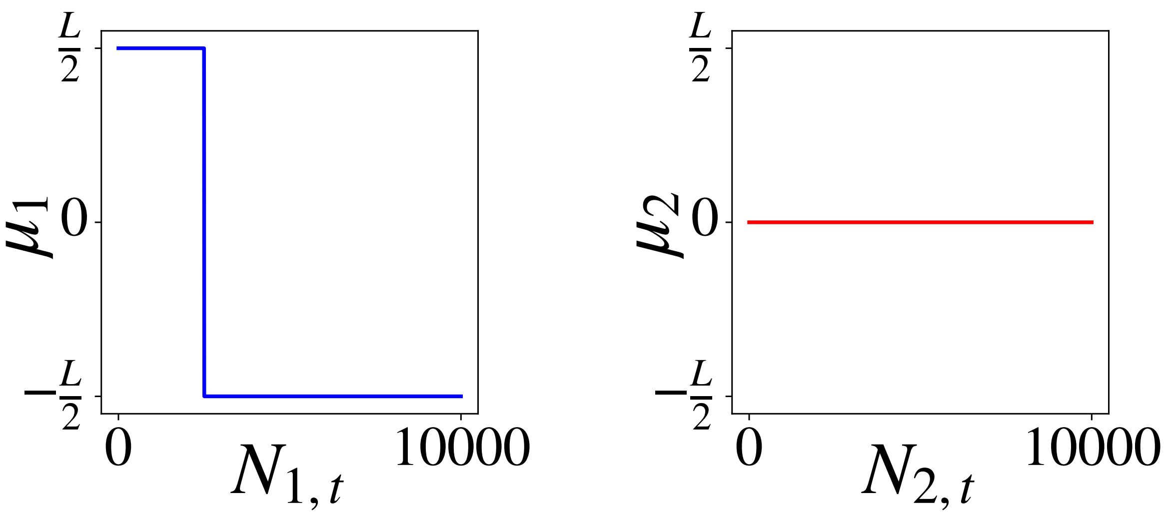
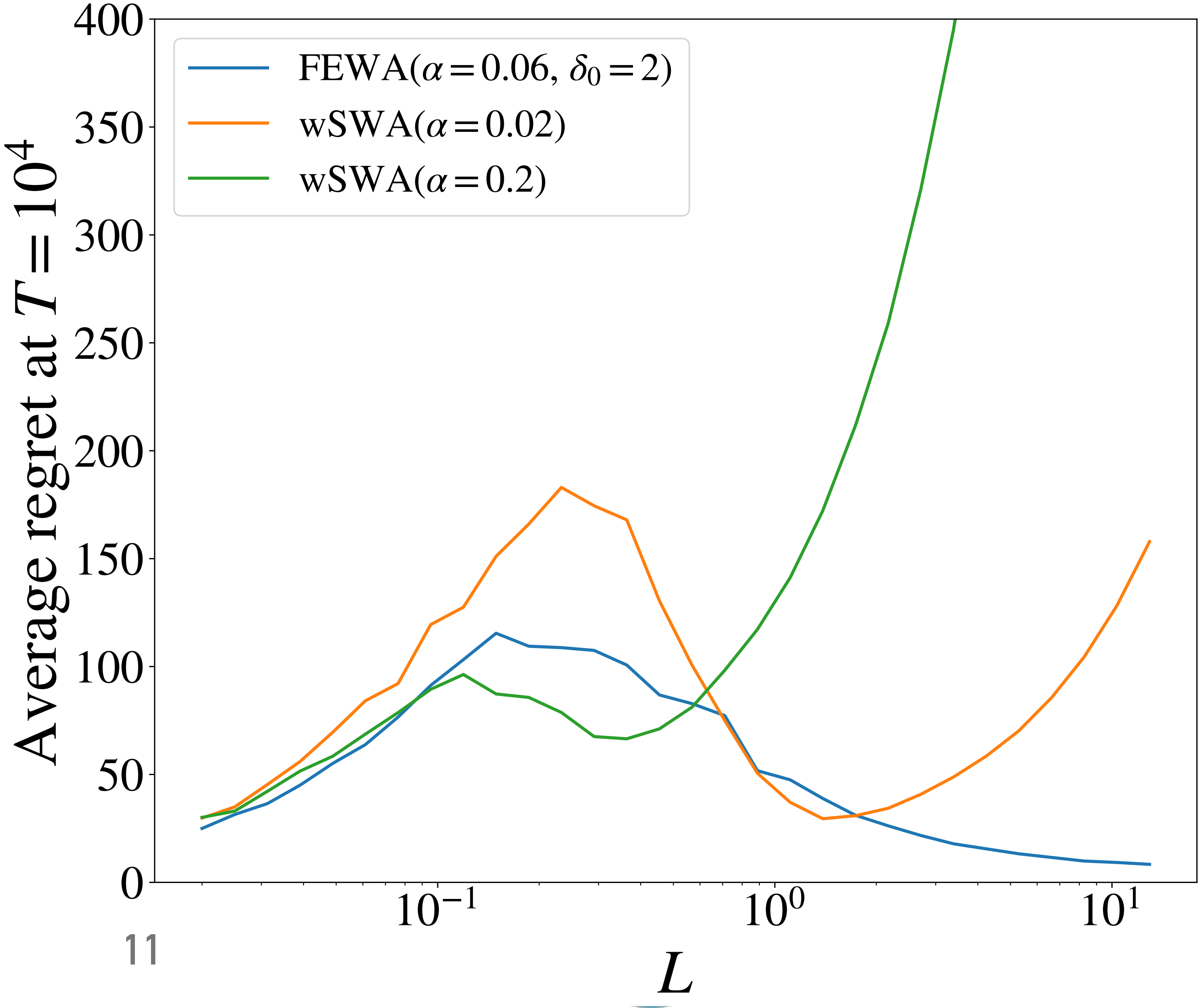


$$\sigma = 1 \; ; \; L$$



CONTRIBUTIONS

1 Rotting bandits are not harder than stochastic bandits

- ✅ $\tilde{\mathcal{O}}\left(\sqrt{KT}\right)$ worst-case bound
- ✅ $\tilde{\mathcal{O}}\left(\log\left(t\right)\right)$ problem-dependent bound

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2 FEWA, a policy

- ✅ with a new data-adaptive window mechanism
- ✅ agnostic to L

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2 FEWA, a policy

- ✅ with a new data-adaptive window mechanism
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3 EFF-FEWA, a policy

- ✅ with FEWA's regret guarantees
- ✅ logarithmic space and time complexity