Graphs in Machine Learning

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Partially based on material by: Mikhail Belkin, Jerry Zhu, Olivier Chapelle, Branislav Kveton

Previous Lecture

- resistive networks
  - recommendation score as a resistance?
  - Laplacian and resistive networks
  - computation of effective resistance
- geometry of the data and the connectivity
- spectral clustering
  - connectivity vs. compactness
  - MinCut, RatioCut, NCut
  - spectral relaxations
- manifold learning

This Lecture

- manifold learning with Laplacian Eigenmaps
- Gaussian random fields and harmonic solution
- Graph-based semi-supervised learning and manifold regularization
- Theory of Laplacian-based manifold methods
- Transductive learning
- SSL Learnability

Previous Lab Session

- 3. 2. 2015 by Daniele.Calandriello@inria.fr
- Content
  - Graph Construction
  - Test sensitivity to parameters: σ, k, ε
  - Spectral Clustering
  - Spectral Clustering vs. k-means
  - Image Segmentation
- Short written report (graded, each lab around 5% of grade)
- Hint: Order 2.1, 2.6 (find the bend), 2.2, 2.3, 2.4, 2.5
- Questions to Daniele.Calandriello@inria.fr
- *Deadline:* 17. 2. 2015
- `http://researchers.lille.inria.fr/~calandri/ta/graphs/td1_handout.pdf`

Advanced Learning for Text and Graph Data

Time: Wednesdays 8h30-11h30 — **4 lectures** and **3 Labs**
Place: Polytechnique / Amphi Sauvy
Lecturer 1: Michalis Vazirgiannis (Polytechnique)
Lecturer 2: Yassine Faihe (Hewlett-Packard - Vertica)

ALTeGraD and **Graphs in ML** run in parallel
The two graph courses are coordinated to be complementary.

Some of covered graph topics not covered in this course

- Ranking algorithms and measures (Kendal Tau, NDCG)
- Advanced graph generators
- Community mining, advanced graph clustering
- Graph degeneracy (k-core & extensions)
- Privacy in graph mining

http://www.math.ens-cachan.fr/version-francaise/formations/master-mva/contenus-/advanced-learning-for-text-and-graph-data-altegrad--239506.kjsp?RH=1242430202531

PhD proposal at CMU and JIE

Joint Institute of Engineering — ***A New Engineering School***

- SYSU-CMU Joint Institute of Engineering (JIE) in Guangzhou, China:
  - International environment, English working language
- Fully-funded PhD positions available at SYSU-CMU JIE:
  - Single-degree program at SYSU in Guangzhou, China
  - Double-degree program (selective)
    - 2 years at CMU, Pittsburgh
    - rest of the time at JIE in Guangzhou, China
- Fundamental research with applications in:
  - Supercomputing & Big Data
  - Biomedical applications
  - Autonomous driving
  - Smart grids and power systems
- Contact: paweng@cmu.edu

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Manifold Learning: Recap

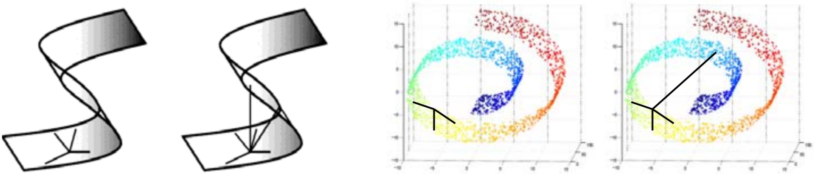
problem: definition reduction/manifold learning

Given $\{\mathbf{x}_i\}_{i=1}^n$ from $\mathbb{R}^d$ find $\{\mathbf{y}_i\}_{i=1}^n$ in $\mathbb{R}^m$, where $m \ll d$.

- What do we know about the **dimensionality reduction**
  - representation/visualization (2D or 3D)
  - an old example: globe to a map
  - often assuming $\mathcal{M} \subset \mathbb{R}^d$
  - feature extraction
  - linear vs. nonlinear dimensionality reduction
- What do we know about linear linear vs. nonlinear methods?
  - linear: ICA, PCA, SVD, ...
  - nonlinear often preserve only **local** distances

Manifold Learning: Linear vs. Non-linear

Manifold Learning: Preserving (just) local distances



$$d(\mathbf{y}_i, \mathbf{y}_j) = d(\mathbf{x}_i, \mathbf{x}_j) \quad \text{only if} \quad d(\mathbf{x}_i, \mathbf{x}_j) \quad \text{is small}$$

$$\min \sum_{ij} w_{ij} \|\mathbf{y}_i - \mathbf{y}_j\|^2$$

Looks familiar?

Manifold Learning: Laplacian Eigenmaps

Step 1: Solve generalized eigenproblem:

$$\mathbf{Lf} = \lambda \mathbf{Df}$$

Step 2: Assign m new coordinates:

$$\mathbf{x}_i \mapsto (f_2(i), \ldots, f_m(i))$$

Note$_1$: we need to get m smallest eigenvectors
Note$_2$: $\mathbf{f}_1$ is useless

http://web.cse.ohio-state.edu/~mbelkin/papers/LEM_NC_03.pdf

Manifold Learning: Laplacian Eigenmaps to 1D

Laplacian Eigenmaps 1D objective

$$\min_{\mathbf{f}} \mathbf{f}^\mathsf{T}\mathbf{L}\mathbf{f} \quad \text{s.t.} \quad f_i \in \mathbb{R}, \quad \mathbf{f}^\mathsf{T}\mathbf{D}\mathbf{1} = 0, \quad \mathbf{f}^\mathsf{T}\mathbf{D}\mathbf{f} = \mathbf{1}$$

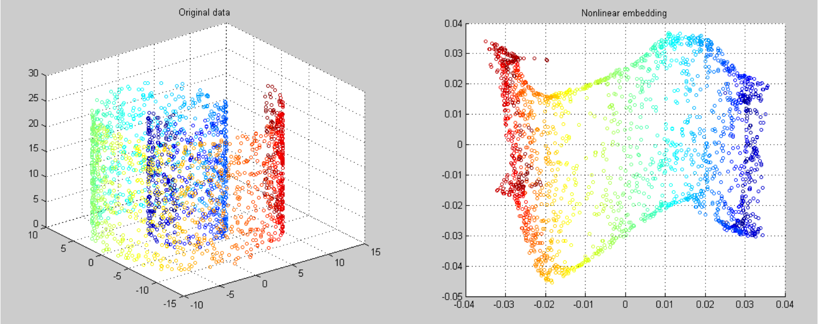
The meaning for constraints is similar as for spectral clustering:

$\mathbf{f}^\mathsf{T}\mathbf{D}\mathbf{f} = \mathbf{1}$ is for scaling

$\mathbf{f}^\mathsf{T}\mathbf{D}\mathbf{1} = 0$ is to not get $\mathbf{v}_1$

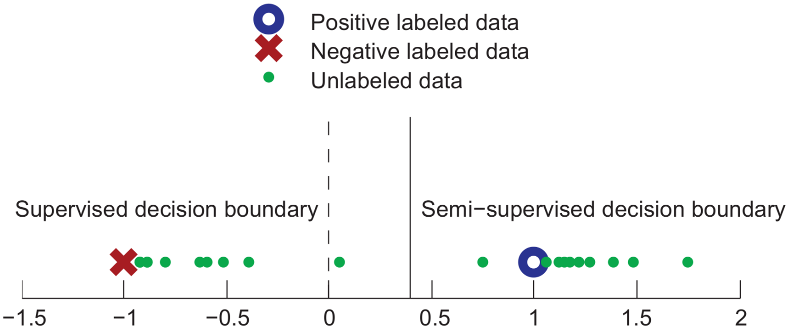
What is the solution?

Manifold Learning: Example



http://www.mathworks.com/matlabcentral/fileexchange/
36141-laplacian-eigenmap-~-diffusion-map-~-manifold-learning

Semi-supervised learning: How is it possible?



This is how children learn! hypothesis

Semi-supervised learning (SSL)

SSL problem: definition

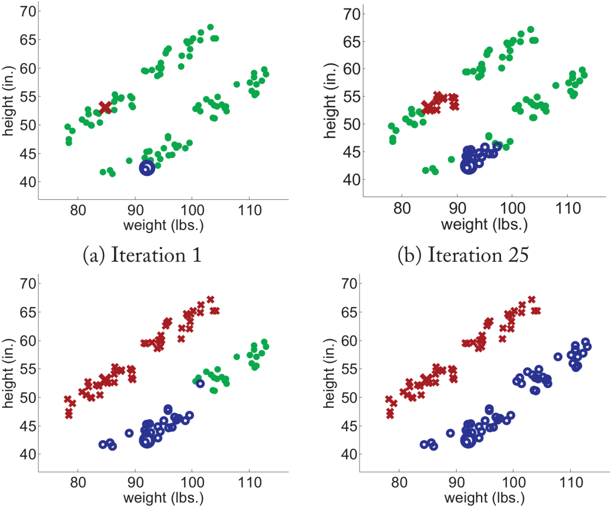
Given $\{\mathbf{x}_i\}_{i=1}^{n}$ from $\mathbb{R}^d$ and $\{y_i\}_{i=1}^{n_l}$, with $n_l \ll n$, find $\{y_i\}_{i=n_l+1}^{n}$ (transductive) or find f predicting y well beyond that (inductive).

Some facts about SSL

- assumes that the unlabeled data is useful
- works with data geometry assumptions
  - cluster assumption - low-density separation
  - manifold assumption
  - smoothness assumptions, generative models, ...
- now it helps now, now it does not (sic)
  - provable cases when it helps
- inductive or transductive/out-of-sample extension

http://olivier.chapelle.cc/ssl-book/discussion.pdf

SSL: Self-Training



(a) Iteration 1

(b) Iteration 25

(c) Iteration 74

(d) Final labeling of all instances

SSL: Overview: Self-Training

SSL: Self-Training

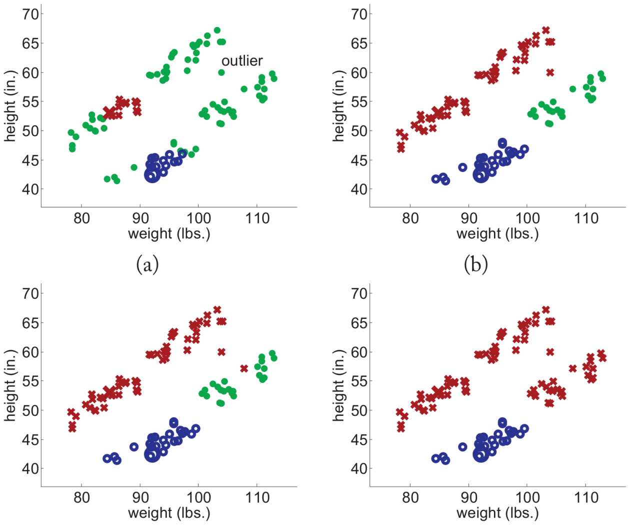
Input: $\mathcal{L} = \{\mathbf{x}_i, y_i\}_{i=1}^{n_l}$ and $\mathcal{U} = \{\mathbf{x}_i\}_{i=n_l+1}^{n}$ **Repeat:**

- train f using $\mathcal{L}$
- apply f to (some) $\mathcal{U}$ and add them to $\mathcal{L}$

What are the properties of self-training?

- its a wrapper method
- heavily depends on the the internal classifier
- some theory exist for specific classifiers
- nobody uses it anymore
- errors propagate (unless the cluster are well separated)

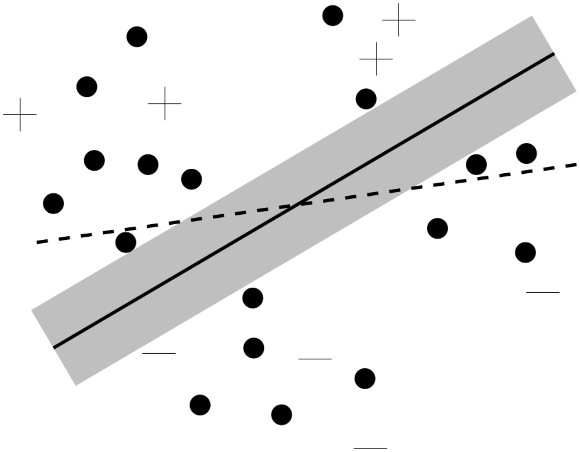
SSL: Self-Training: Bad Case



(a) (b)

(c) (d)

SSL: Transductive SVM: S3VM



SSL: Transductive SVM: Classical SVM

Linear case: $f = \mathbf{w}^\mathsf{T}\mathbf{x} + b \quad \rightarrow \quad$ we look for $(\mathbf{w}, b)$

max-margin classification

$$\max_{\mathbf{w},b} \quad \frac{1}{\|\mathbf{w}\|}$$
$$s.t. \quad y_i(\mathbf{w}^\mathsf{T}\mathbf{x}_i + b) \geq 1 \quad \forall i = 1, \dots, n_l$$

max-margin classification

$$\min_{\mathbf{w},b} \quad \|\mathbf{w}\|^2$$
$$s.t. \quad y_i(\mathbf{w}^\mathsf{T}\mathbf{x}_i + b) \geq 1 \quad \forall i = 1, \dots, n_l$$

SSL: Transductive SVM: Classical SVM

max-margin classification: separable case

$$
\begin{aligned}
\min_{\mathbf{w},b} \quad & \|\mathbf{w}\|^2 \\
s.t. \quad & y_i(\mathbf{w}^\mathsf{T}\mathbf{x}_i + b) \geq 1 \quad \forall i = 1, \dots, n_l
\end{aligned}
$$

max-margin classification: non-separable case

$$
\begin{aligned}
\min_{\mathbf{w},b} \quad & \lambda\|\mathbf{w}\|^2 + \sum_i \xi_i \\
s.t. \quad & y_i(\mathbf{w}^\mathsf{T}\mathbf{x}_i + b) \geq 1 - \xi_i \quad \forall i = 1, \dots, n_l \\
& \xi_i \geq 0 \quad \forall i = 1, \dots, n_l
\end{aligned}
$$

SSL: Transductive SVM: Classical SVM

max-margin classification: non-separable case

$$\min_{\mathbf{w},b} \quad \lambda\|\mathbf{w}\|^2 + \sum_i \xi_i$$

$$s.t. \quad y_i(\mathbf{w}^\mathsf{T}\mathbf{x}_i + b) \geq 1 - \xi_i \quad \forall i = 1, \dots, n_l$$

$$\xi_i \geq 0 \quad \forall i = 1, \dots, n_l$$

Unconstrained formulation:

$$\min_{\mathbf{w},b} \sum_i^l \max\left(1 - y_i\left(\mathbf{w}^\mathsf{T}\mathbf{x}_i + b\right), 0\right) + \lambda\|\mathbf{w}\|^2$$

In general?

$$\min_{\mathbf{w},b} \sum_i^l V(\mathbf{x}_i, y_i, f(\mathbf{x}_i)) + \lambda\Omega(f)$$

SSL: Transductive SVM: Unlabeled Examples

$$\min_{\mathbf{w},b} \sum_{i}^{n_l} \max\left(1 - y_i\left(\mathbf{w}^\mathsf{T}\mathbf{x}_i + b\right), 0\right) + \lambda\|\mathbf{w}\|^2$$

How to incorporate unlabeled examples?

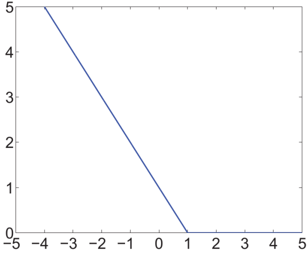
No y's for unlabeled $\mathbf{x}$.

Prediction of f for (any) $\mathbf{x}$? $\widehat{y} = \operatorname{sgn}(f(\mathbf{x})) = \operatorname{sgn}(\mathbf{w}^\mathsf{T}\mathbf{x} + b)$

Pretending that $\operatorname{sgn}(f(\mathbf{x}))$ is true ...

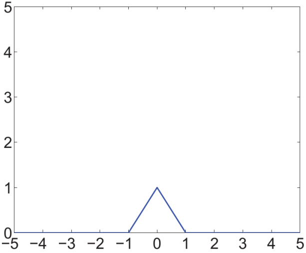
$$\begin{aligned} V(\mathbf{x}, \hat{y}, f(\mathbf{x})) &= \max\left(1 - \hat{y}\left(\mathbf{w}^\mathsf{T}\mathbf{x} + b\right), 0\right) \\ &= \max\left(1 - \operatorname{sgn}\left(\mathbf{w}^\mathsf{T}\mathbf{x} + b\right)\left(\mathbf{w}^\mathsf{T}\mathbf{x} + b\right), 0\right) \\ &= \max\left(1 - \left|\mathbf{w}^\mathsf{T}\mathbf{x} + b\right|, 0\right) \end{aligned}$$

SSL: Transductive SVM: Hinge and Hat Loss



$yf(\mathbf{x})$

(a) the hinge loss



$f(\mathbf{x})$

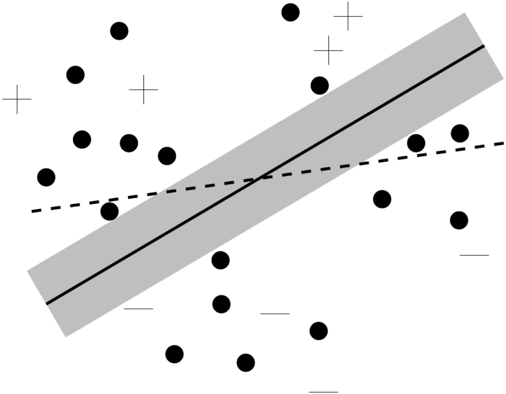
(b) the hat loss

What is the difference in the objectives?

Hinge loss penalizes? the margin of being on the wrong side

Hat loss penalizes? predicting in the margin

SSL: Transductive SVM: S3VM



This what we wanted!

SSL: Transductive SVM: Formulation

Main SVM idea stays: penalize the margin

$$\min_{\mathbf{w},b} \sum_{i=1}^{n_l} \max\left(1 - y_i\left(\mathbf{w}^\mathsf{T}\mathbf{x}_i + b\right), 0\right) + \lambda_1 \|\mathbf{w}\|^2 + \lambda_2 \sum_{i=l+1}^{n_l+n_u} \max\left(1 - \left|\mathbf{w}^\mathsf{T}\mathbf{x}_i + b\right|, 0\right)$$

What is the loss and what is the regularizer?

$$\min_{\mathbf{w},b} \sum_{i=1}^{n_l} \max\left(1 - y_i\left(\mathbf{w}^\mathsf{T}\mathbf{x}_i + b\right), 0\right) + \lambda_1 \|\mathbf{w}\|^2 + \lambda_2 \sum_{i=l+1}^{n_l+n_u} \max\left(1 - \left|\mathbf{w}^\mathsf{T}\mathbf{x}_i + b\right|, 0\right)$$

Think of **unlabeled data** as the **regularizers** for your classifiers!

Practical hint: Additionally enforce the class balance.

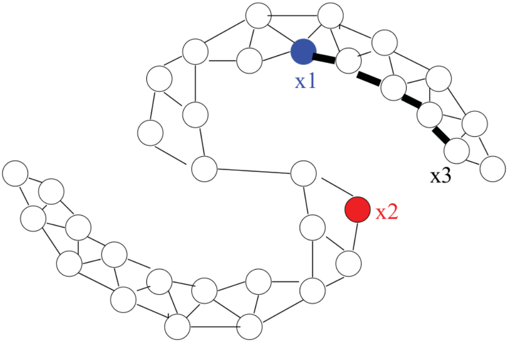
Another problem: Optimization is difficult.

SSL with Graphs: Prehistory

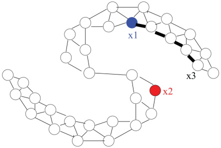
Blum/Chawla: Learning from Labeled and Unlabeled Data using Graph Mincuts

`http://www.aladdin.cs.cmu.edu/papers/pdfs/y2001/mincut.pdf`

*following some insights from vision research in 1980s



SSL with Graphs: MinCut



MinCut SSL: an idea similar to MinCut clustering

Where is the link? connected classes, not necessarily compact

What is the formal statement? We look for $f(\mathbf{x}) \in \{\pm 1\}$

$$\mathrm{cut} = w_{ij} \sum_{i,j=1}^{n_l+n_u} (f(\mathbf{x}_i) - f(\mathbf{x}_j))^2 = \Omega(f)$$

Why $(f(\mathbf{x}_i) - f(\mathbf{x}_j))^2$ and not $|f(\mathbf{x}_i) - f(\mathbf{x}_j)|$? It does not matter.

SSL with Graphs: MinCut

We look for $f(\mathbf{x}) \in \{\pm 1\}$

$$\Omega(f) = \sum_{i,j=1}^{n_l+n_u} w_{ij} \left(f(\mathbf{x}_i) - f(\mathbf{x}_j)\right)^2$$

Clustering was unsupervised, here we have supervised data.

Recall the general objective framework:

$$\min_{\mathbf{w},b} \sum_i^l V(\mathbf{x}_i, y_i, f(\mathbf{x}_i)) + \lambda \Omega(f)$$

It would be nice if we match the prediction on labeled data:

$$V(\mathbf{x}, y, f(\mathbf{x})) = \infty \sum_{i=1}^{n_l} (f(\mathbf{x}) - y)^2$$

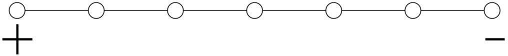
SSL with Graphs: MinCut

Final objective function:

$$\min_{f \in \{\pm 1\}^{n_l+n_u}} \infty \sum_{i=1}^{n_l} (f(\mathbf{x}) - y)^2 + \lambda \sum_{i,j=1}^{n_l+n_u} w_{ij} \left(f(\mathbf{x}_i) - f(\mathbf{x}_j)\right)^2$$

This is an integer program :(

Can we solve it? It still just MinCut. Are we happy?



There are six solutions. All equivalent.

We need a better way to reflect the confidence.

SSL with Graphs: Harmonic Functions

Zhu/Ghahramani/Lafferty: Semi-Supervised Learning Using Gaussian Fields and Harmonic Functions

http://mlg.eng.cam.ac.uk/zoubin/papers/zgl.pdf

*a seminal paper that convinced people to use graphs for SSL

Idea 1: Look for a **unique** solution.

Idea 2: Find a smooth one. (**Harmonic** solution)

Harmonic SSL

1): As before we constrain f to match the supervised data:

$$f(\mathbf{x}_i) = y_i \qquad \forall i \in \{1, \dots, n_l\}$$

2): We enforce the solution f to be harmonic.

$$f(\mathbf{x}_i) = \frac{\sum_{i \sim j} f(\mathbf{x}_j) w_{ij}}{\sum_{i \sim j} w_{ij}} \qquad \forall i \in \{n_l + 1, \dots, n_u + n_l\}$$

SSL with Graphs: Harmonic Functions

The harmonic solution is obtained from the mincut one ...

$$\min_{f \in \{\pm 1\}^{n_l+n_u}} \infty \sum_{i=1}^{n_l} (f(\mathbf{x}_i) - y_i)^2 + \lambda \sum_{i,j=1}^{n_l+n_u} w_{ij} \left(f(\mathbf{x}_i) - f(\mathbf{x}_j)\right)^2$$

... if we just relax the integer constraints to be real ...

$$\min_{f \in \mathbb{R}^{n_l+n_u}} \infty \sum_{i=1}^{n_l} (f(\mathbf{x}_i) - y_i)^2 + \lambda \sum_{i,j=1}^{n_l+n_u} w_{ij} \left(f(\mathbf{x}_i) - f(\mathbf{x}_j)\right)^2$$

... or equivalently (note that $f(\mathbf{x}_i) = f_i$) ...

$$\min_{f \in \mathbb{R}^{n_l+n_u}} \sum_{i,j=1}^{n_l+n_u} w_{ij} \left(f(\mathbf{x}_i) - f(\mathbf{x}_j)\right)^2$$

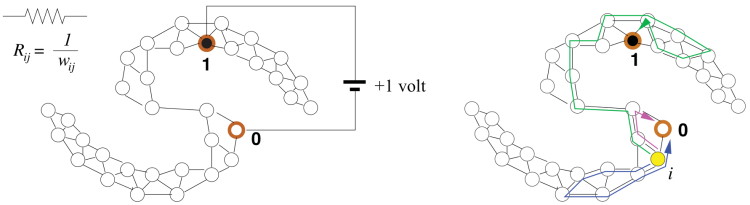
$$s.t. \quad y_i = f(\mathbf{x}_i) \quad \forall i = 1, \ldots, n_l$$

SSL with Graphs: Harmonic Functions

Properties of the relaxation from ± 1 to $\mathbb{R}$

- there is a closed form solution for f
- this solution is unique
- globally optimal
- it is either constant or has a maximum /minimum on a boundary
- $f(\mathbf{x}_i)$ may not be discrete
  - but we can threshold it
- random walk interpretation
- electric networks interpretation

SSL with Graphs: Harmonic Functions



(a) The electric network interpretation (b) The random walk interpretation

Random walk interpretation:

1) start from the vertex to label and follow

2) $P(j|i) = \frac{w_{ij}}{\sum_k w_{ik}} \qquad \equiv \qquad \mathbf{P} = \mathbf{D}^{-1}\mathbf{W}$

3) finish when the labeled vertex is hit

absorbing random walk

f_i = probability of reaching a positive labeled vertex

SSL with Graphs: Harmonic Functions

How to compute HS? **Option A:** iteration/propagation

Step 1: Set $f(\mathbf{x}_i) = y_i$ for $i = 1, \ldots, n_l$
Step 2: Propagate iteratively (only for unlabeled)

$$f(\mathbf{x}_i) \leftarrow \frac{\sum_{i\sim j} f(\mathbf{x}_j) w_{ij}}{\sum_{i\sim j} w_{ij}} \qquad \forall i \in \{n_l + 1, \ldots, n_u + n_l\}$$

Properties:

- this will converge to the harmonic solution
- we can set the initial values for unlabeled nodes arbitrarily
- an interesting option for large-scale data

SSL with Graphs: Harmonic Functions

How to compute HS? **Option B:** Closed form solution

Define $\mathbf{f} = (f(\mathbf{x}_1), \ldots, f(\mathbf{x}_{n_l+n_u})) = (f_1, \ldots, f_{n_l+n_u})$

$$\Omega(f) = \sum_{i,j=1}^{n_l+n_u} w_{ij} \left(f(\mathbf{x}_i) - f(\mathbf{x}_j)\right)^2 = \mathbf{f}^\mathsf{T}\mathbf{L}\mathbf{f}$$

$\mathbf{L}$ is a $(n_l + n_u) \times (n_l + n_u)$ matrix:

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}_{ll} & \mathbf{L}_{lu} \\ \mathbf{L}_{u1} & \mathbf{L}_{uu} \end{bmatrix}$$

How to compute this **constrained** minimization problem?

Yes, Lagrangian multipliers are an option, but . . .

SSL with Graphs: Harmonic Functions

Let us compute **harmonic** solution using **harmonic** property!

How did we formalize the harmonic property of a circuit?

$$(\mathbf{Lf})_u = \mathbf{0}$$

In matrix notation

$$\begin{bmatrix} \mathbf{L}_{ll} & \mathbf{L}_{lu} \\ \mathbf{L}_{ul} & \mathbf{L}_{uu} \end{bmatrix} \begin{bmatrix} \mathbf{f}_l \\ \mathbf{f}_u \end{bmatrix} = \begin{bmatrix} \mathbf{0}_l \\ \mathbf{0}_u \end{bmatrix}$$

$\mathbf{f}_l$ is constrained to be $\mathbf{y}_l$ and for $\mathbf{f}_u$ ......

$$\mathbf{L}_{ul}\mathbf{f}_l + \mathbf{L}_{uu}\mathbf{f}_u = \mathbf{0}_u$$

...from which we get

$$\mathbf{f}_u = \mathbf{L}_{uu}^{-1}(-\mathbf{L}_{ul}\mathbf{f}_l) = \mathbf{L}_{uu}^{-1}(\mathbf{W}_{ul}\mathbf{f}_l).$$

SSL with Graphs: Harmonic Functions

Can we see that this calculate the probability of a random walk?

$$\mathbf{f}_u = \mathbf{L}_{uu}^{-1}(-\mathbf{L}_{ul}\mathbf{f}_l) = \mathbf{L}_{uu}^{-1}(\mathbf{W}_{ul}\mathbf{f}_l)$$

Note that $\mathbf{P} = \mathbf{D}^{-1}\mathbf{W}$. Then equivalently

$$\mathbf{f}_u = (\mathbf{I} - \mathbf{P}_{uu})^{-1}\mathbf{P}_{ul}\mathbf{f}_l.$$

Split the equation into +ve & -ve part:

$$\begin{aligned} f_i &= (\mathbf{I} - \mathbf{P}_{uu})_{iu}^{-1}\mathbf{P}_{ul}\mathbf{f}_l \\ &= \underbrace{\sum_{j:y_j=1} (\mathbf{I} - \mathbf{P}_{uu})_{iu}^{-1}\mathbf{P}_{uj}}_{p_i^{(+1)}} - \underbrace{\sum_{j:y_j=-1} (\mathbf{I} - \mathbf{P}_{uu})_{iu}^{-1}\mathbf{P}_{uj}}_{p_i^{(-1)}} \\ &= p_i^{(+1)} - p_i^{(-1)} \end{aligned}$$

SSL with Graphs: Regularized Harmonic Functions

$$f_i = \underbrace{|f_i|}_{\text{confidence}} \times \underbrace{\mathrm{sgn}(f_i)}_{\text{label}}$$

What if a nasty outlier sneaks in?

The prediction for the outlier can be hyperconfident :(

How to control the confidence of the inference?

Allow the random walk to **die**!

We add a **sink** to the graph.

sink = artificial label node with value 0

We connect it to every other vertex.

What will this do to our predictions?

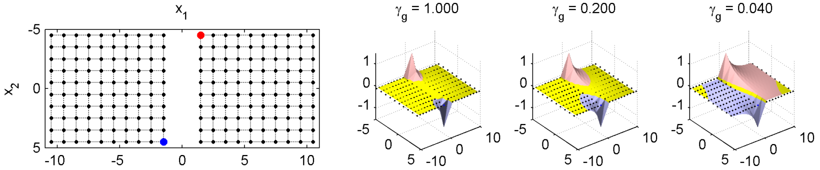
depends on the weigh on the edges

SSL with Graphs: Regularized Harmonic Functions

How do we compute this **regularized** random walk?

$$\mathbf{f}_u = (\mathbf{L}_{uu} + \gamma_g \mathbf{I})^{-1} (\mathbf{W}_{ul} \mathbf{f}_l)$$

How does γ_g influence HS?



What happens to sneaky outliers?

SSL with Graphs: Soft Harmonic Functions

Regularized HS objective with $\mathbf{Q} = \mathbf{L} + \gamma_g \mathbf{I}$:

$$\min_{\mathbf{f}\in\mathbb{R}^{n_l+n_u}} \infty \sum_{i=1}^{n_l} w_{ij}\left(f(\mathbf{x}_i) - y_i\right)^2 + \lambda \mathbf{f}^\mathsf{T}\mathbf{Q}\mathbf{f}$$

What if we do not really believe that $f(\mathbf{x}_i) = y_i, \forall i$?

$$\mathbf{f}^\star = \min_{\mathbf{f}\in\mathbb{R}^n} (\mathbf{f}-\mathbf{y})^\mathsf{T}\mathbf{C}(\mathbf{f}-\mathbf{y}) + \mathbf{f}^\mathsf{T}\mathbf{Q}\mathbf{f}$$

$\mathbf{C}$ is diagonal with $C_{ii} = \begin{cases} c_l & \text{for labeled examples} \\ c_u & \text{otherwise.} \end{cases}$

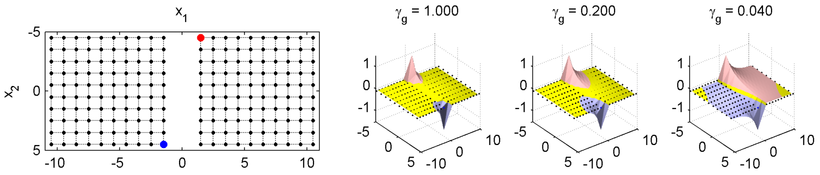
$\mathbf{y} \equiv$ pseudo-targets with $y_i = \begin{cases} \text{true label} & \text{for labeled examples} \\ 0 & \text{otherwise.} \end{cases}$

SSL with Graphs: Soft Harmonic Functions

$$\mathbf{f}^\star = \min_{\mathbf{f}\in\mathbb{R}^n} (\mathbf{f}-\mathbf{y})^\mathsf{T}\mathbf{C}(\mathbf{f}-\mathbf{y}) + \mathbf{f}^\mathsf{T}\mathbf{Q}\mathbf{f}$$

Closed form **soft harmonic solution**:

$$\mathbf{f}^\star = (\mathbf{C}^{-1}\mathbf{Q}+\mathbf{I})^{-1}\mathbf{y}$$



What are the differences between hard and soft?

Not much different in practice.

Provable generalization guarantees for soft.

SSL with Graphs: Regularized Harmonic Functions

Larger implications of random walks

random walk relates to **commute distance** which should satisfy

($\star$) Vertices in the same cluster of the graph have a small commute distance, whereas two vertices in different clusters of the graph have a "large" commute distance.

Do we have this property for HS? What if $n \to \infty$?

Luxburg/Radl/Hein: *Getting lost in space: Large sample analysis of the commute distance* `http://www.informatik.uni-hamburg.de/ML/contents/people/luxburg/publications/LuxburgRadlHein2010_PaperAndSupplement.pdf`

Solutions? 1) γ_g 2) amplified commute distance 3) $\mathbf{L}^p$ 4) $\mathbf{L}^\star$ ...

The goal of these solutions: **make them remember!**

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