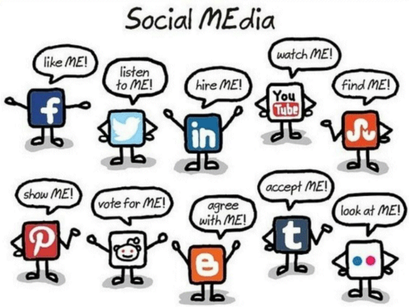
Graphs in Machine Learning

Michal Valko

INRIA Lille - Nord Europe, France

Graphs from **social networks**

- people and their interactions
- directed (Twitter) and undirected (Facebook)
- structure is rather a *phenomena*
- typical ML tasks
  - advertising
  - product placement
  - link prediction (PYMK)



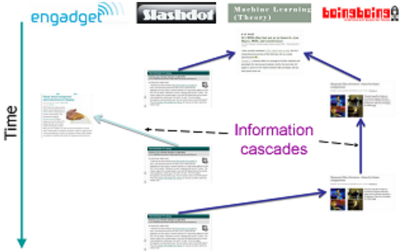
Graphs from **utility and technology networks**

- link services
- power grids, roads, Internet, sensor networks
- structure is either *hand designed* or not
- typical ML tasks
  - best routing under unknown or variable costs
  - identify the node of interest

Berkley's Floating Sensor Network

Graphs from **information networks**

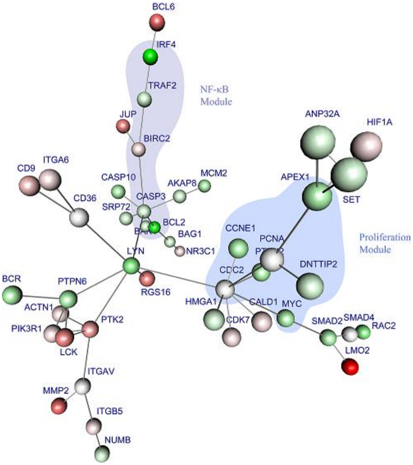
- web
- blogs
- wikipedia
- typical ML tasks
  - find influential sources
  - search (pagerank)



Blogs cascades (ETH) - *submodularity*

Graphs from **biological networks**

- protein-protein interactions
- gene regulatory networks
- typical ML tasks
  - discover unexplored interactions
  - learn or reconstruct the structure



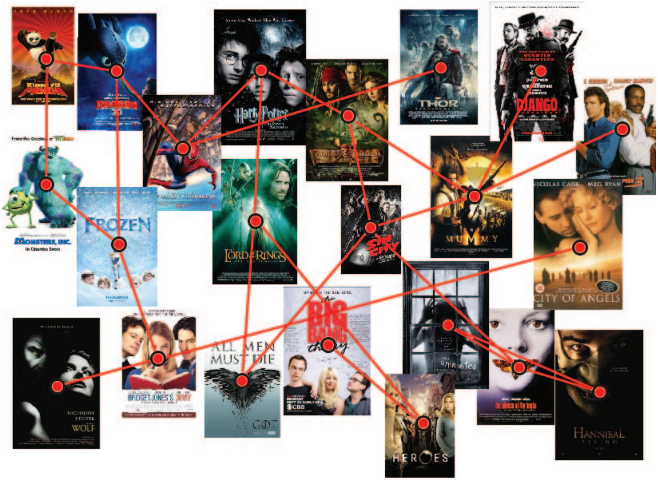
Diffuse large B-cell lymphomas - Dittrich et al. (2008)

Graphs from **similarity networks**

graph is not naturally given

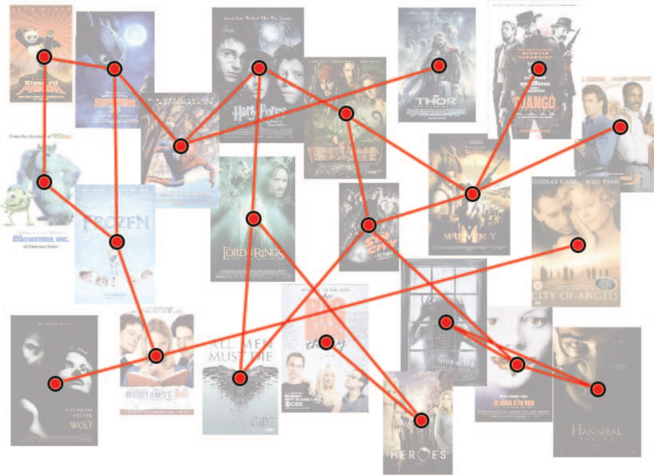
Graphs from **similarity networks**

but we can construct it



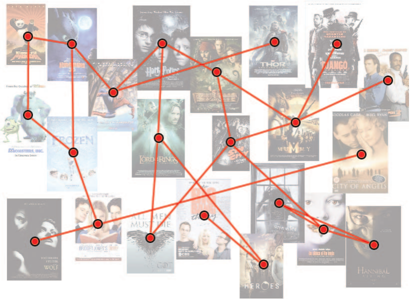
Graphs from **similarity networks**

and use it as an abstraction



Graphs from **similarity networks**

- vision
- audio
- text
- typical ML tasks
  - semi-supervised learning
  - spectral clustering



Movie similarity

Two sources of graphs in ML

Graph as models for networks

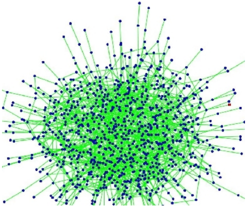
- given as an input
- discover interesting properties of the structure
- represent useful information (viral marketing)
- be the object of study (anomaly detection)

Graph as nonparametric basis

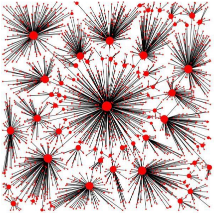
- we create (learn) the structure
- flat vectorial data $\rightarrow$ similarity graph
- nonparametric regularizer
- encode structural properties: smoothness, independence, ...

Random Graph Models

Erdős-Rényi
independent edges



Barabási-Albert
preferential attachment



Stochastic Blocks
modeling communities

Watts-Strogatz, Chung-Lu, Fiedler, ...

What will you learn in the Graphs in ML course?

Concepts, tools, and methods to work with graphs in ML.

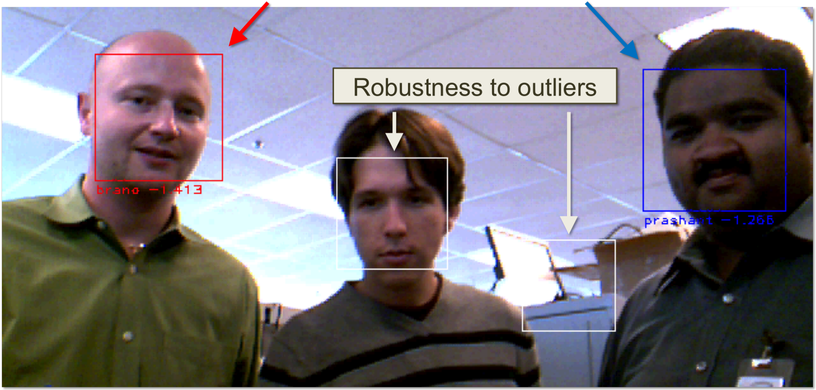
Theoretical toolbox to analyze graph based algorithms.

Specific applications of graphs in ML.

One example: Online Semi-Supervised Face Recognition

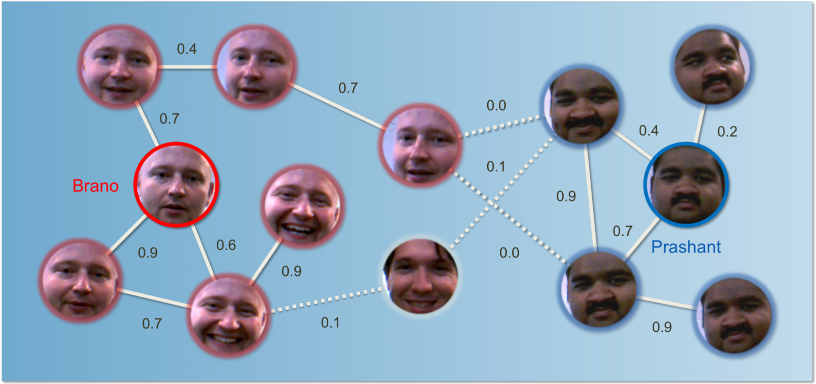
Online Semi-Supervised Face Recognition

graph is not given



Online Semi-Supervised Face Recognition

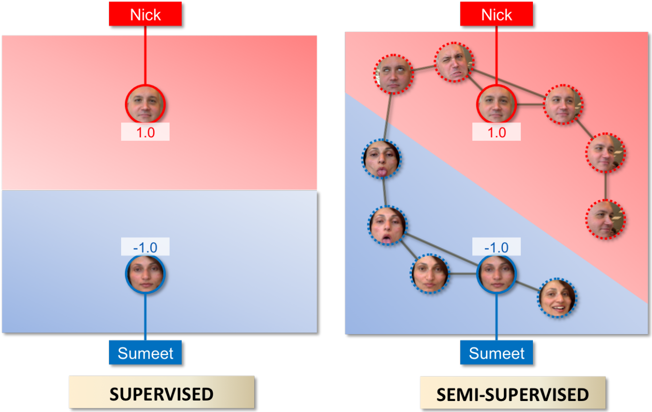
we will construct it!



An example of a similarity graph over faces. The faces are vertices of the graph. The edges of the graph connect similar faces. Labeled faces are outlined by thick solid lines.

Online Semi-Supervised Face Recognition

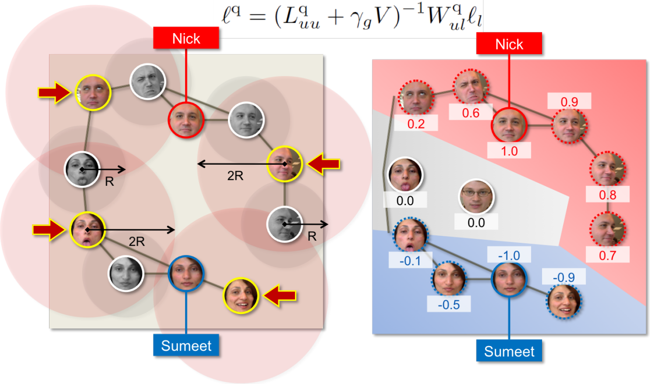
graph-based semi-supervised learning



Online Semi-Supervised Face Recognition

online learning - graph sparsification

$$\ell^{q} = (L^{q}_{uu} + \gamma_g V)^{-1} W^{q}_{ul} \ell_l$$



DEMO

second TD

see the demo: http://researchers.lille.inria.fr/~valko/hp/serve.php?what=publications/kveton2009nipsdemo.officespace.mov

OSS FaceReco: Analysis

$$\frac{1}{n}\sum_t(\ell_t^{\mathrm{q}}[t]-y_t)^2 \le \frac{3}{n}\sum_t(\ell_t^{*}-y_t)^2 + \frac{3}{n}\sum_t(\ell_t^{\mathrm{o}}[t]-\ell_t^{*})^2 + \frac{3}{n}\sum_t(\ell_t^{\mathrm{q}}[t]-\ell_t^{\mathrm{o}}[t])^2$$

Error of our solution	Offline learning error	Online learning error	Quantization error

Claim: When the regularization parameter is set as $\gamma_g = \Omega(n_l^{3/2})$, the difference between the risks on labeled and all vertices decreases at the rate of $O(n_l^{-1/2})$ (with a high probability)

$$\frac{1}{n}\sum_t(\ell_t^{*}-y_t)^2 \le \frac{1}{n_l}\sum_{i\in l}(\ell_i^{*}-y_i)^2 + \beta + \sqrt{\frac{2\ln(2/\delta)}{n_l}}(n_l\beta+4)$$

$$\beta \le \left[\frac{\sqrt{2}}{\gamma_g+1} + \sqrt{2n_l}\,\frac{1-\sqrt{c_u}}{\sqrt{c_u}}\,\frac{\lambda_M(L)+\gamma_g}{\gamma_g^2+1}\right]$$

OSS FaceReco: Analysis

$$\frac{1}{n}\sum_t (\ell_t^{\mathrm{q}}[t]-y_t)^2 \le \frac{3}{n}\sum_t (\ell_t^* - y_t)^2 + \frac{3}{n}\sum_t (\ell_t^{\mathrm{o}}[t]-\ell_t^*)^2 + \frac{3}{n}\sum_t (\ell_t^{\mathrm{q}}[t]-\ell_t^{\mathrm{o}}[t])^2$$

Error of our solution	Offline learning error	Online learning error	Quantization error

Claim: When the regularization parameter is set as $\gamma_g = \Omega(n^{1/4})$, the average error between the offline and online HFS predictions decreases at the rate of $O(n^{-1/2})$

$$\frac{1}{n}\sum_t (\ell_t^{\mathrm{o}}[t]-\ell_t^*)^2 \le \frac{1}{n}\sum_t \left\|\ell^{\mathrm{o}}[t]-\ell^*\right\|_2^2 \le \frac{4n_l}{(\gamma_g+1)^2}$$

$$\|\ell\|_2 \le \frac{\|y\|_2}{\lambda_m(C^{-1}K+I)} = \frac{\|y\|_2}{\lambda_m(K)\lambda_M^{-1}(C)+1} \le \frac{\sqrt{n_l}}{\gamma_g+1}$$

OSS FaceReco: Analysis

$$\frac{1}{n}\sum_t (\ell_t^{\mathrm{q}}[t]-y_t)^2 \le \frac{3}{n}\sum_t (\ell_t^{*}-y_t)^2 + \frac{3}{n}\sum_t (\ell_t^{\mathrm{o}}[t]-\ell_t^{*})^2 + \frac{3}{n}\sum_t (\ell_t^{\mathrm{q}}[t]-\ell_t^{\mathrm{o}}[t])^2$$

Error of our solution

Offline learning error

Online learning error

Quantization error

Claim: When the regularization parameter is set as $\gamma_g = \Omega(n^{1/8})$, and the Laplacians L^q and L^o and normalized, the average error between the online and online quantized HFS predictions decreases at the rate of $O(n^{-1/2})$

$$\frac{1}{n}\sum_t (\ell_t^{\mathrm{q}}[t]-\ell_t^{\mathrm{o}}[t])^2 \le \frac{1}{n}\sum_t \left\|\ell^{\mathrm{q}}[t]-\ell^{\mathrm{o}}[t]\right\|_2^2 \le \frac{n_l}{c_u^2\gamma_g^4}\left\|L^{\mathrm{q}}-L^{\mathrm{o}}\right\|_F^2$$

$$\left\|L^{\mathrm{q}}-L^{\mathrm{o}}\right\|_F^2 \propto O(k^{-2/d})$$

The distortion rate of online k-center clustering is $O(k^{-1/d})$, where d is dimension of the manifold and k is the number of representative vertices

Some of the other topics

- link prediction/link clasification
- signed networks (eOpinions)
- online decision-making on graphs
- submodularity on graphs
- real world graphs scalability and approximations
- spectral sparsification
- recommender systems applications
- large graph analysis, learning, and mining
- generalization bounds by perturbation analysis

Time: Tuesdays 11h-13h

Place: ENS Cachan - Room C103 (Bldg. Cournot, 1. floor)

8 lectures: 13.1. 20.1. 27.1. 10.2. 17.2. 3.3. 17.3. 24.3.

3 recitations (TD): 3.2. 24.2. 10.3.

Validation: grades from TD + class project

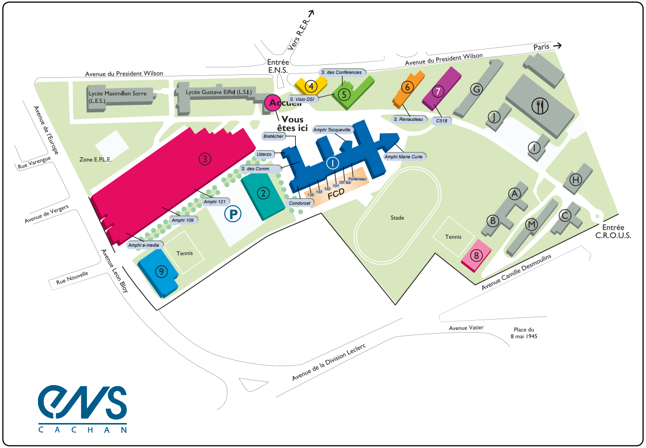
Research: contact me for *internships*, *PhD. theses*, *projects*, etc.

Course website:
`http://researchers.lille.inria.fr/~valko/hp/mva-ml-graphs`

Contact:
Lecturer: Michal.Valko @ inria.fr

TA: Daniele.Calandriello @ inria.fr

See you on Tuesday, January 13th, at 11h!



Michal Valko
michal.valko@inria.fr
sequel.lille.inria.fr