Graphs in Machine Learning

Large-Scale Machine Learning on Graphs

Computational Bottlenecks and Challenges

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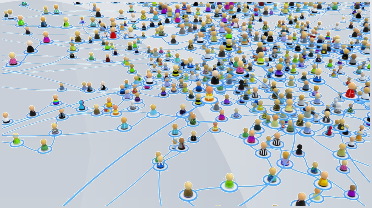
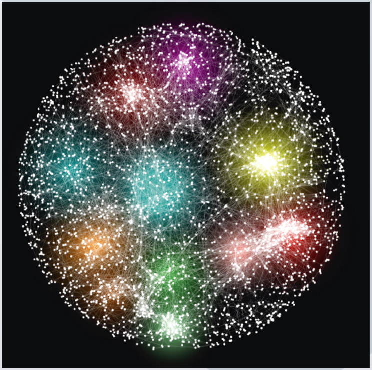
Partially based on material by: Rob Fergus, Nikhil Srivastava, Yiannis Koutis, Joshua Batson, Daniel Spielman

Large scale Machine Learning on Graphs

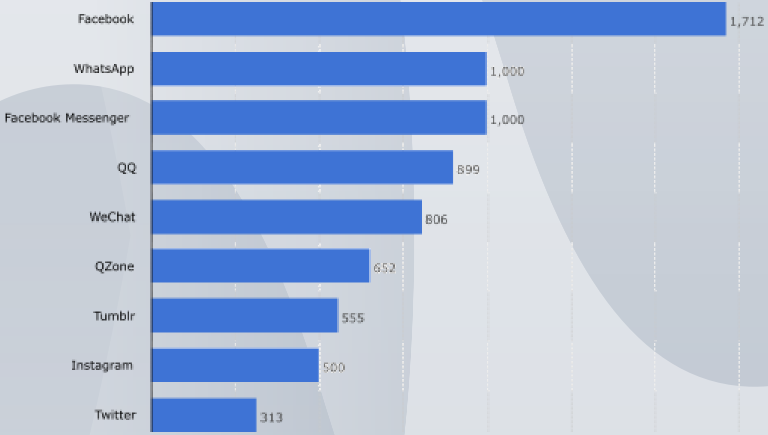


http://blog.carsten-eickhoff.com



Botstein et al.

Are we large yet?



"One **trillion** edges: graph processing at Facebook-scale."
Ching et al., VLDB 2015

Computational bottlenecks

In theory:

Space

$[\mathcal{O}(m), \mathcal{O}(n^2)]$ to store

Time

$\mathcal{O}(n^2)$ to construct

$\mathcal{O}(n^3)$ to run algorithms

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- 2012 Common Crawl Corpus:

  3.5 Billion pages (45 GB)

  128 Billion edges (331 GB)

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  128 Billion edges (331 GB)
- Pagerank on Facebook Graph:
  3 minutes per iteration, hundreds of iterations, tens of hours on 200 machines, run once per day

Two phases

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2. Run your algorithm on the graph

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Can we find close neighbours without checking all distances?

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Split your data in small subset of close points

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More general problem: learning good codeword representation

Storing graph in memory

Main bottleneck: **space**.

As a Fermi (back-of-the-envelope) problem

- Storing a graph with m edges require to store m tuples $(i, j, w_{i,j})$ of 64 bit (8 bytes) doubles or int.

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- The largest compute-optimized cloud instances have 64$+$ cores, but only 128 GB of memory.
- We can store $128 * 1024^3/(3 * 8) \sim 5.7 \times 10^9$ (5.7 billion) edges in a single machine memory.

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  More on this later

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https://misovalko.github.io/mva-ml-graphs.html