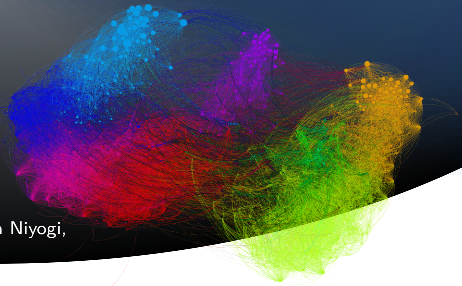
Graphs in Machine Learning

Semi-Supervised Learning Introduction

Why and When SSL Helps

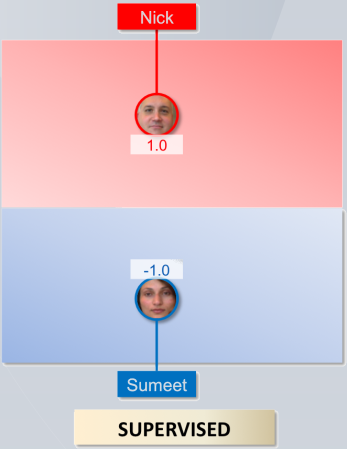
Michal Valko

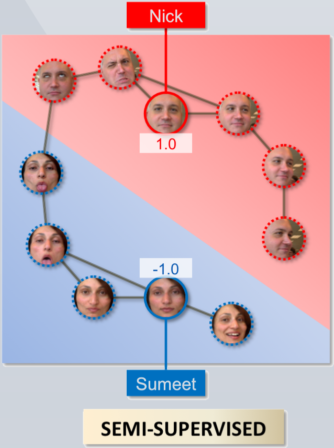
Inria & ENS Paris-Saclay, MVA



material by: Mikhail Belkin, Partha Niyogi,
Schölkopf

Semi-supervised learning: How is it possible?





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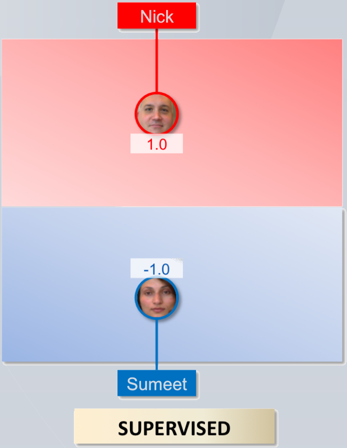

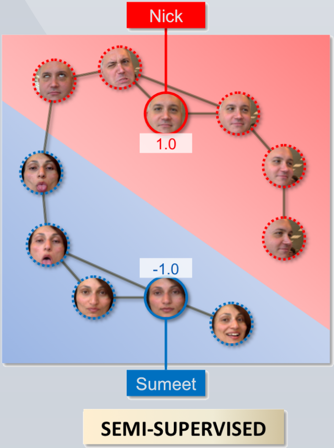

This is how children learn! hypothesis

Semi-supervised learning (SSL)

SSL problem: definition

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- inductive or transductive/out-of-sample extension

http://olivier.chapelle.cc/ssl-book/discussion.pdf

SSL: Overview: Self-Training

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Repeat:

- train f using $\mathcal{L}$
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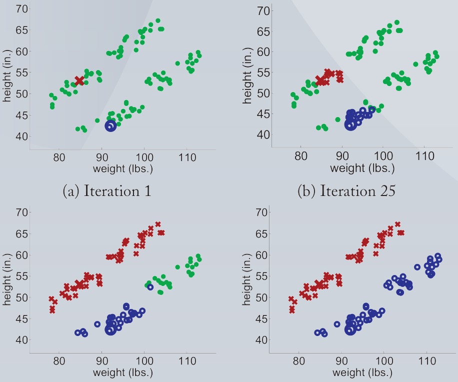
- errors propagate (unless the clusters are well separated)

SSL: Self-Training (Good Case)

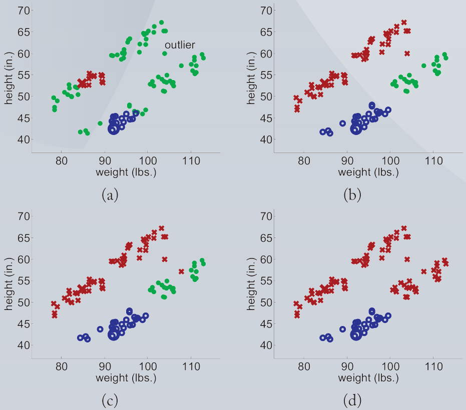


(a) Iteration 1 (b) Iteration 25

(c) Iteration 74 (d) Final labeling of all instances

Source: **chapelle2006semi-supervised<empty citation>**

SSL: Self-Training (Bad Case)



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SSL($\mathcal{G}$)

semi-supervised learning with graphs and harmonic functions

...our running example for learning with graphs

SSL with Graphs: Prehistory

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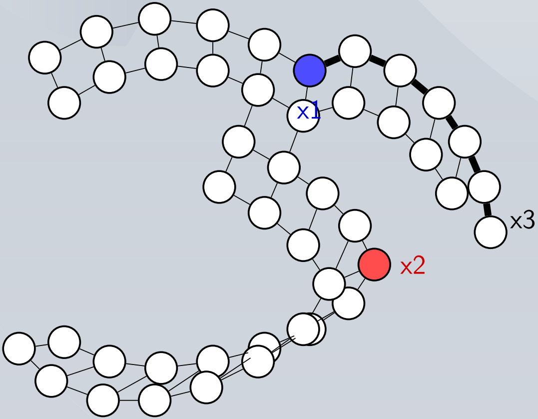
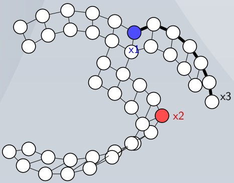


SSL with Graphs: MinCut



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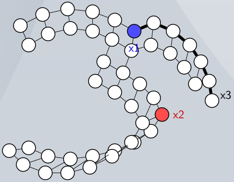

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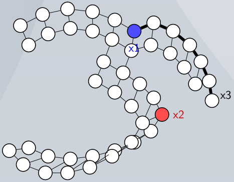

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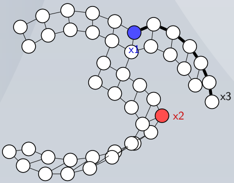

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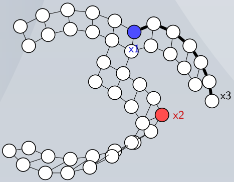

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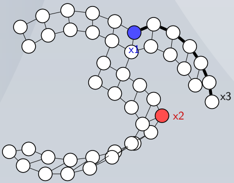

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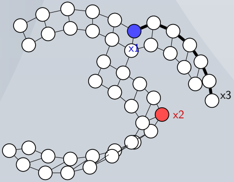

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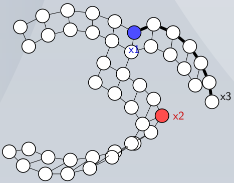

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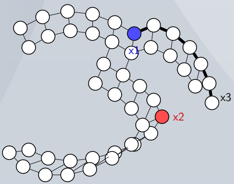

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Why $\left(f\left(\mathbf{x}_i\right) - f\left(\mathbf{x}_j\right)\right)^2$ and not $|f(\mathbf{x}_i) - f(\mathbf{x}_j)|$? It does not matter.

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SSL with Graphs: MinCut

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Final objective function:

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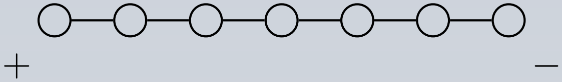

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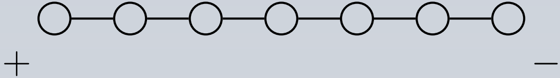

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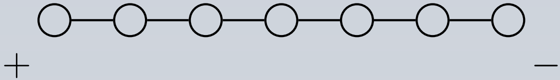

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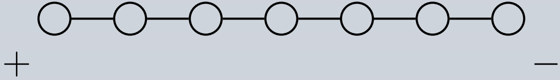

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We need a better way to reflect the confidence.

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Inria & ENS Paris-Saclay, MVA

https://misovalko.github.io/mva-ml-graphs.html