Graphs in Machine Learning

Resistive Networks

Physics and Effective Resistance

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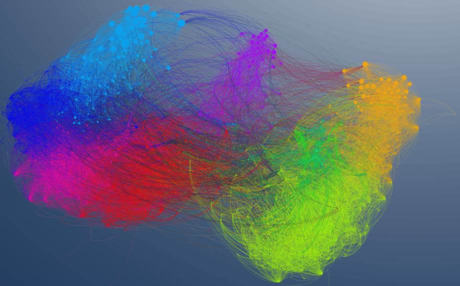
Partially based on material by: Ulrike von Luxburg, Gary Miller, Doyle & Schnell, Daniel Spielman



Laplacians and Resistive Networks

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Idea$_4$: view edges as conductors

$\mathrm{score}_4(v, m) =$ effective resistance between m and v

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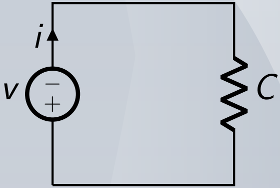

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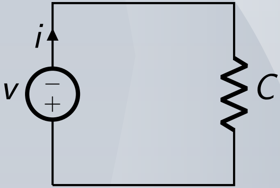

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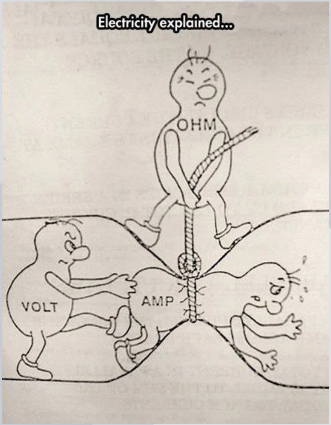
$$C = \frac{1}{R} \qquad i = CV = \frac{V}{R}$$

Resistive Networks: Some high-school physics



Source: Illustration commonly attributed to Vagonum

Resistive Networks

Resistive Networks

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Effective Resistance on a graph

Take two nodes: $a \neq b$. Let V_{ab} be the voltage between them and i_{ab} the current between them. Define $R_{ab} = \frac{V_{ab}}{i_{ab}}$ and $C_{ab} = \frac{1}{R_{ab}}$.

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We treat the entire graph as a resistor!

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The effective resistance is a distance!

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https://misovalko.github.io/mva-ml-graphs.html