Graphs in Machine Learning

Spectral Clustering: Graph Cuts

MinCut, RatioCut, and NCut

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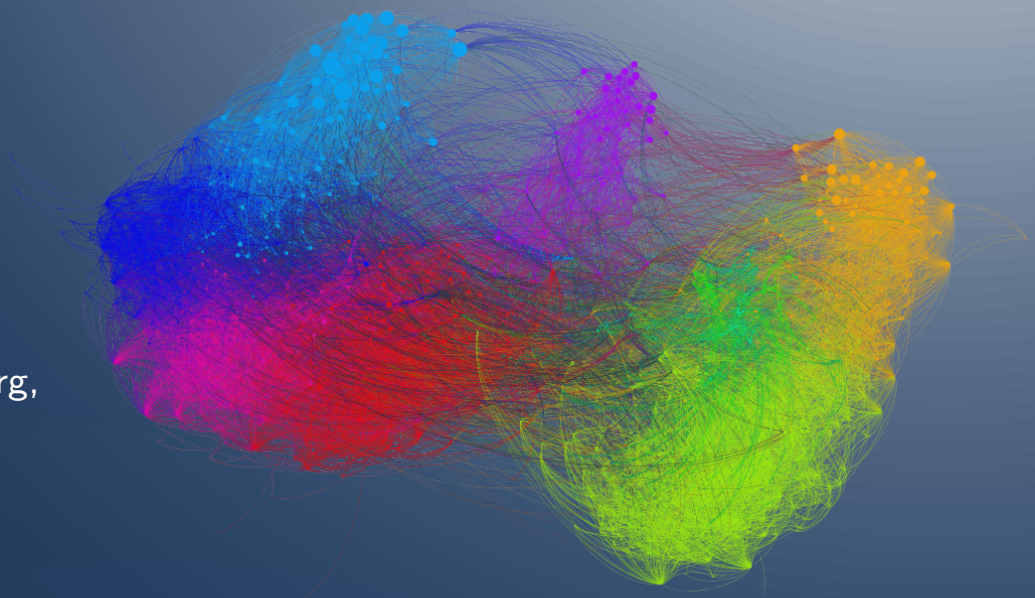
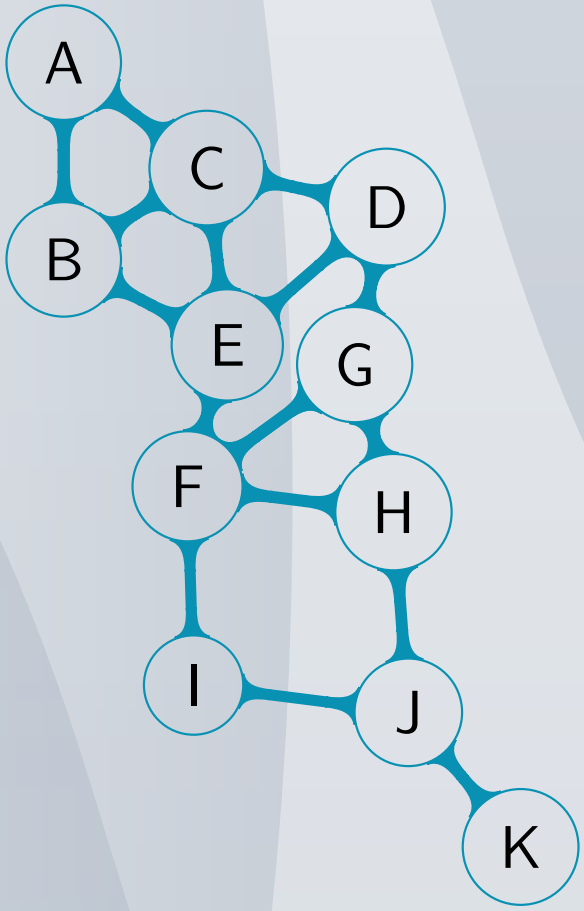
Partially based on material by: Ulrike von Luxburg, Gary Miller, Doyle & Schnell, Daniel Spielman



Spectral Clustering: Cuts on graphs



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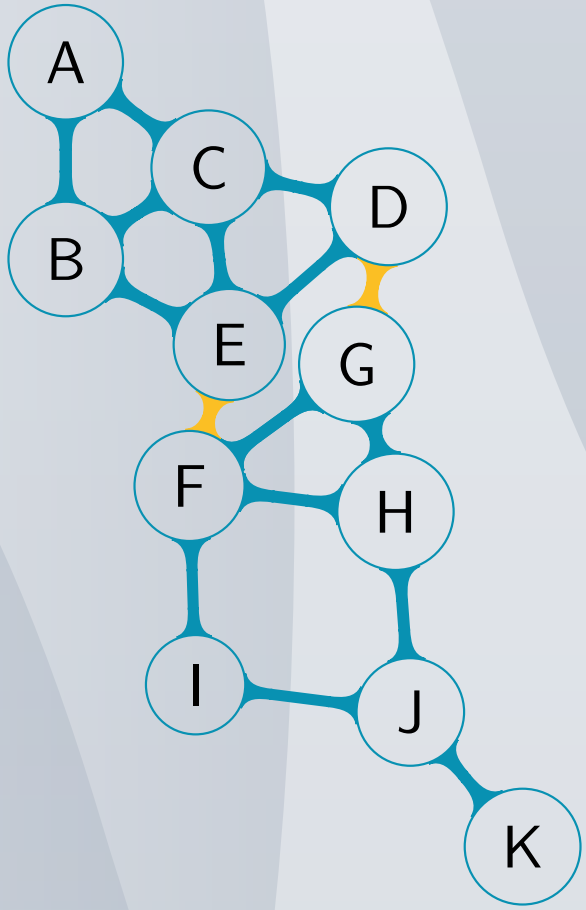

Defining the cut objective we get the clustering!

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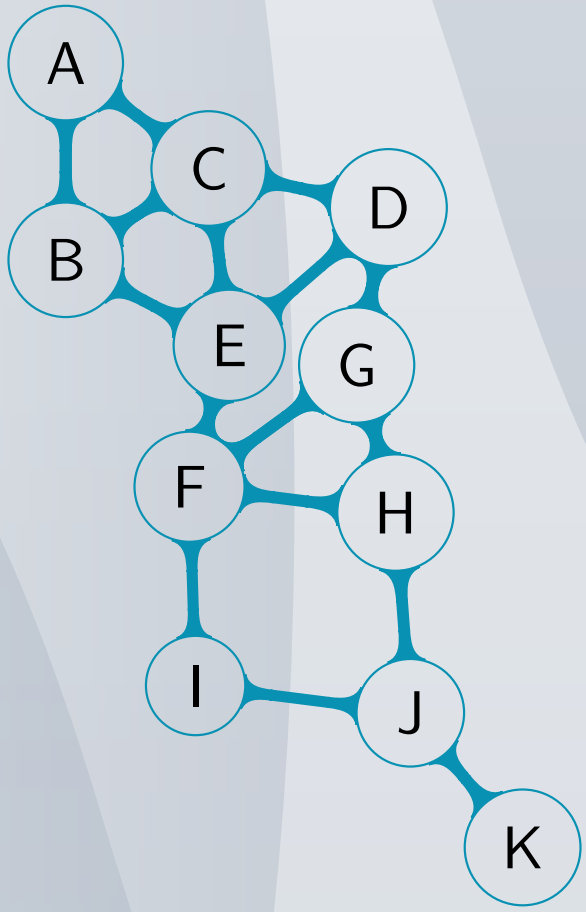

MinCut: $\mathrm{cut}(A, B) = \sum_{i \in A, j \in B} w_{ij}$

Are we done?

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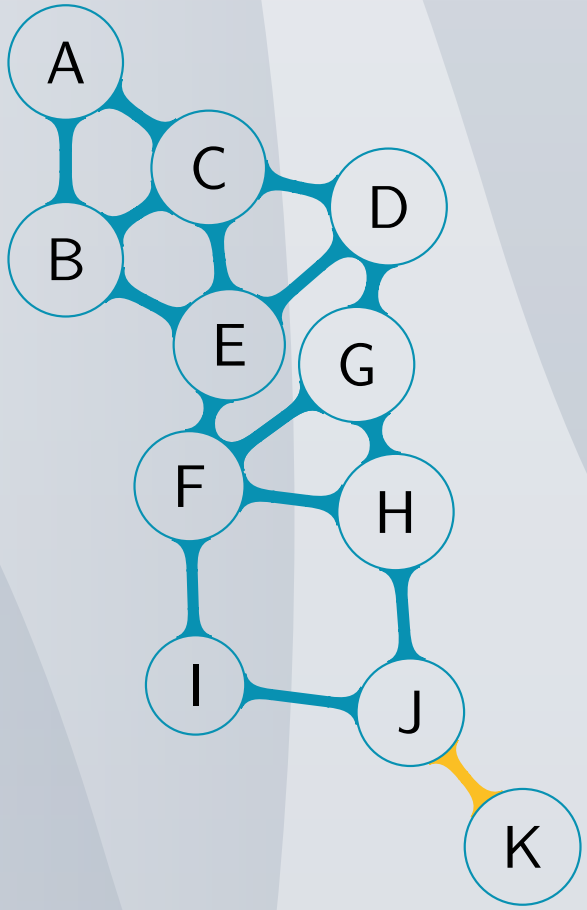

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Are we done?

Can be solved efficiently, but maybe not what we want . . . .

Spectral Clustering: Balanced Cuts

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Normalized Cut

$$\mathrm{NCut}(A, B) = \sum_{i \in A, j \in B} w_{ij} \left(\frac{1}{\mathrm{vol}(A)} + \frac{1}{\mathrm{vol}(B)} \right)$$

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Easily generalizable to $k \geq 2$

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Approximate!

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https://misovalko.github.io/mva-ml-graphs.html