Graphs in Machine Learning

Online SSL: Graph Quantization

Charikar's Algorithm and Multiplicities

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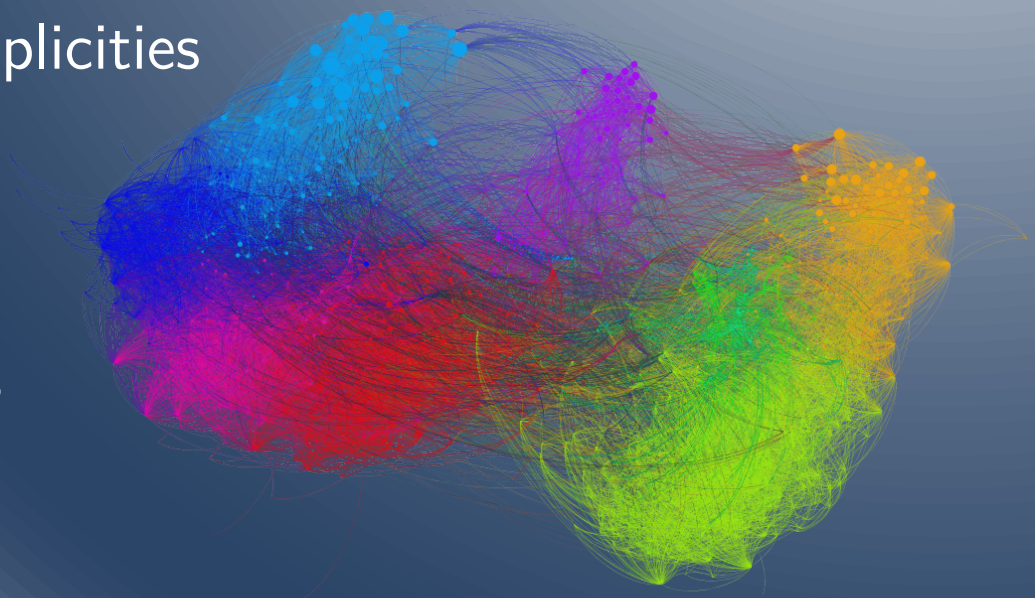
Partially based on material by: Branislav Kveton, Mikhail Belkin, Jerry Zhu



Online SSL with Graphs: Graph Quantization

An idea: incremental k-centers

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Doubling algorithm of Charikar et al. [**charikar1997incremental**]

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- if not possible, R is doubled

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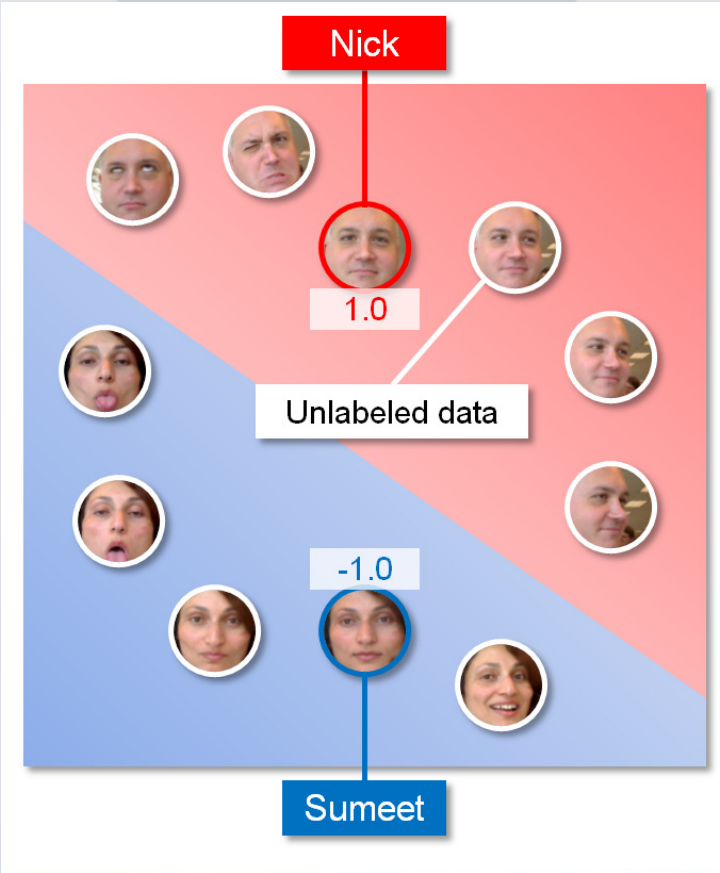

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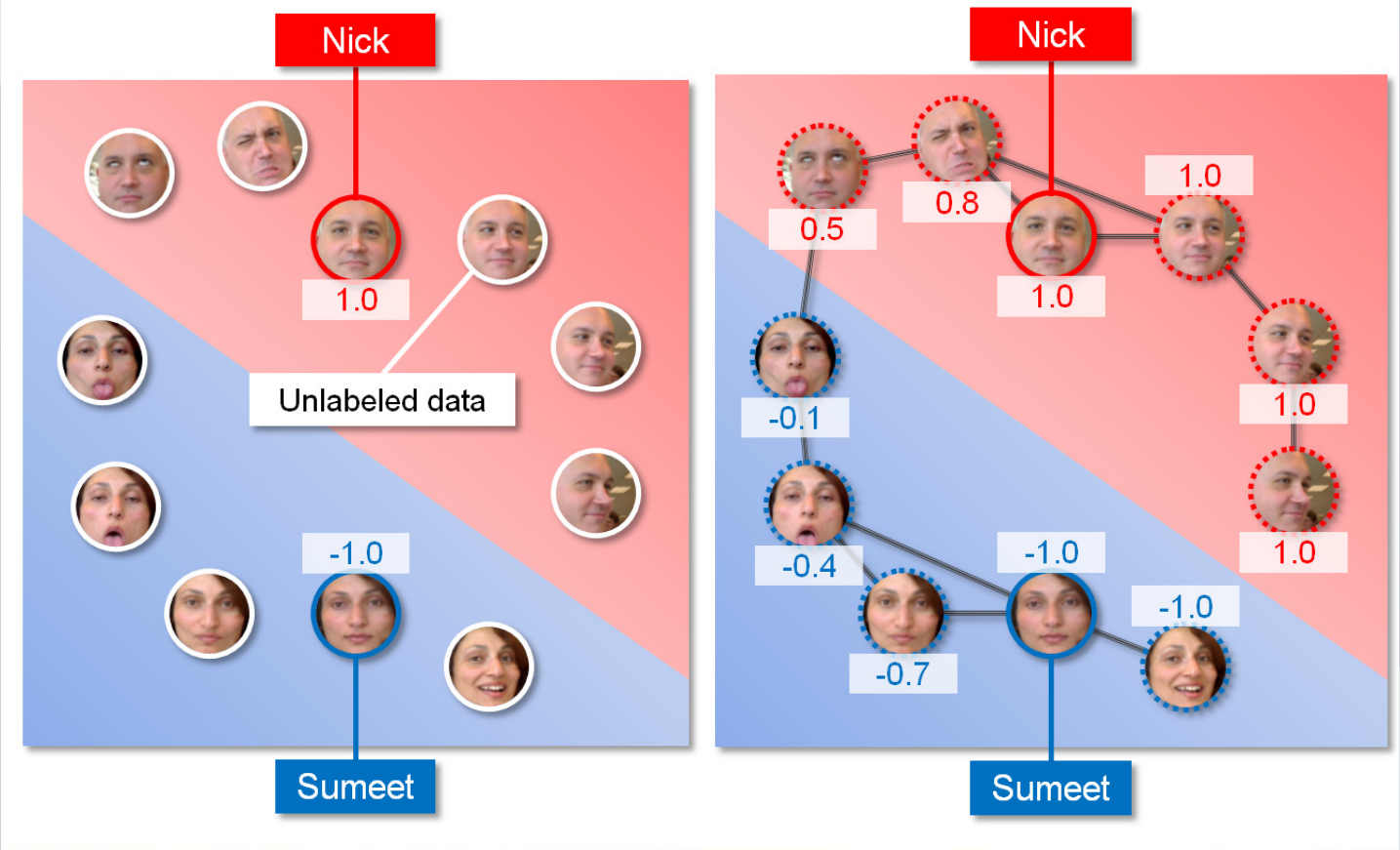

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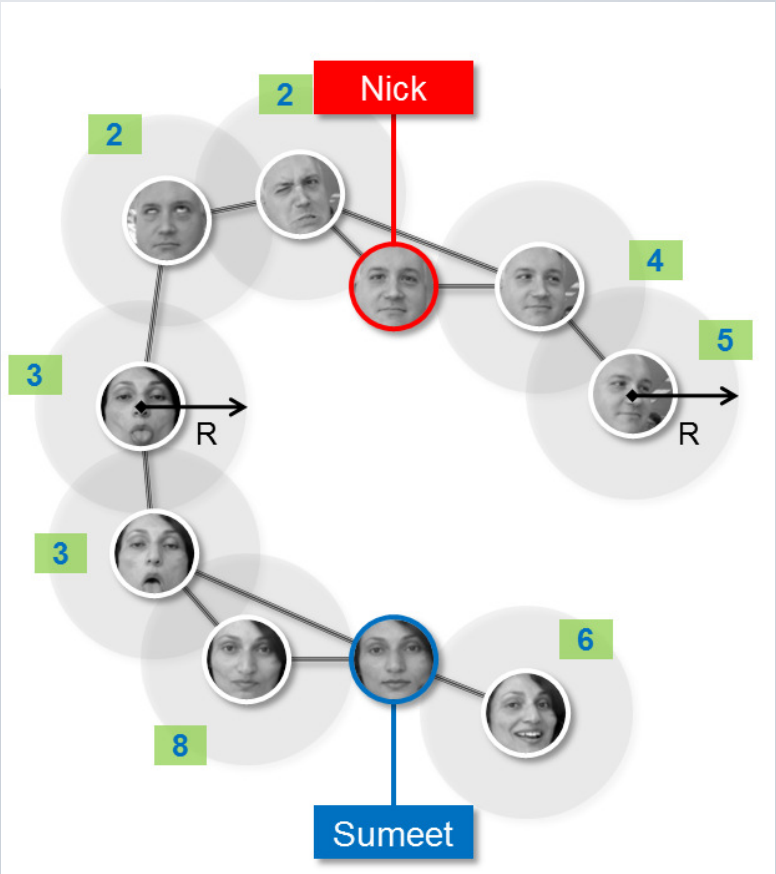

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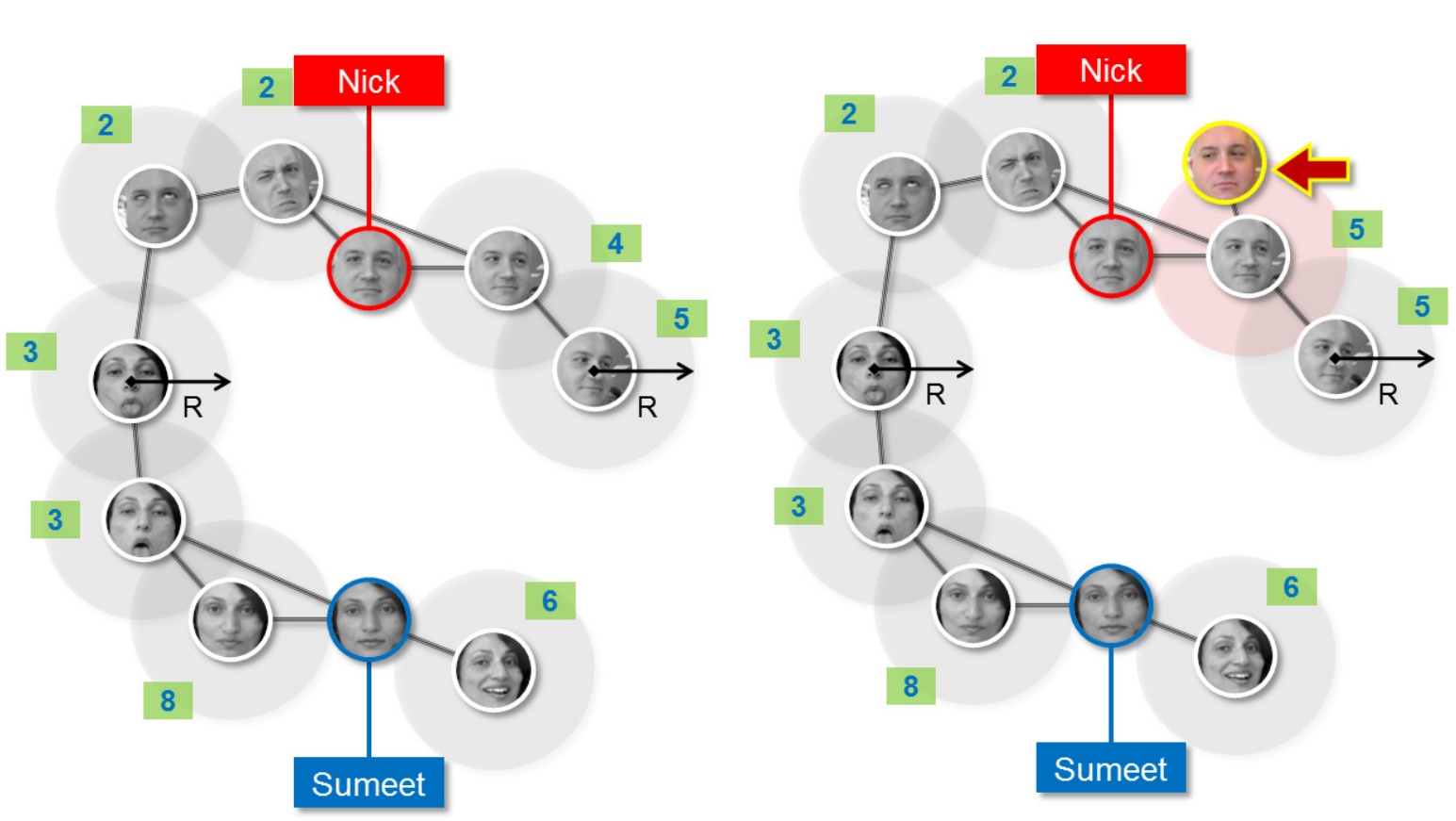

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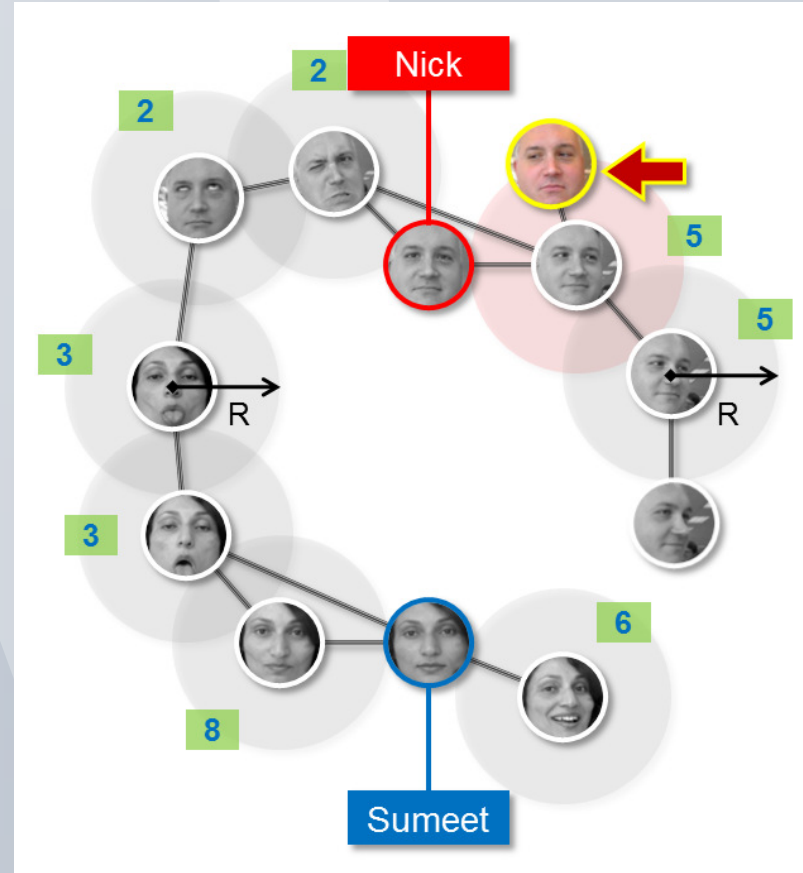

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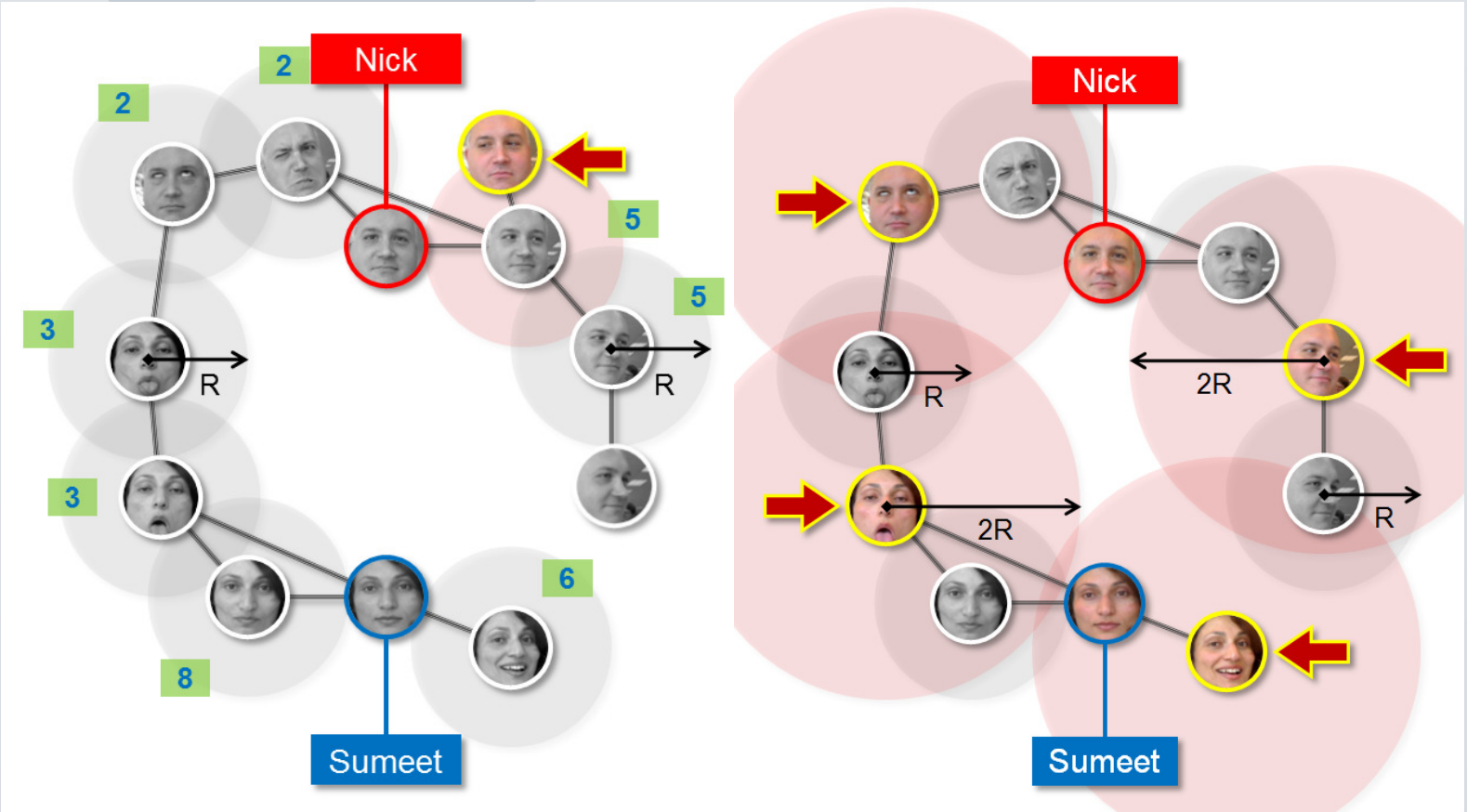

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Online k-centers

1: an unlabeled $\mathbf{x}_t$, a set of centroids C_{t-1}, multiplicities $\mathbf{v}_{t-1}$

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2: **if** $(|C_{t-1}| = k + 1)$ **then**
3: $\quad R \leftarrow mR$
4: $\quad$ greedily repartition C_{t-1} into C_t such that:
5: $\quad\quad$ no two vertices in C_t are closer than R
6: $\quad\quad$ for any $\mathbf{c}_i \in C_{t-1}$ exists $\mathbf{c}_j \in C_t$ such that $d(\mathbf{c}_i, \mathbf{c}_j) < R$

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an unlabeled $\mathbf{x}_t$, a set of centroids C_{t-1}, multiplicities $\mathbf{v}_{t-1}$
if $(|C_{t-1}| = k + 1)$ **then**
    $R \leftarrow mR$
    greedily repartition C_{t-1} into C_t such that:
        no two vertices in C_t are closer than R
        for any $\mathbf{c}_i \in C_{t-1}$ exists $\mathbf{c}_j \in C_t$ such that $d(\mathbf{c}_i, \mathbf{c}_j) < R$
    update $\mathbf{v}_t$ to reflect the new partitioning
else
    $C_t \leftarrow C_{t-1}$
    $\mathbf{v}_t \leftarrow \mathbf{v}_{t-1}$

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else
    $C_t \leftarrow C_{t-1}$
    $\mathbf{v}_t \leftarrow \mathbf{v}_{t-1}$
end if
if $\mathbf{x}_t$ is closer than R to any $\mathbf{c}_i \in C_t$ **then**
    $\mathbf{v}_t(i) \leftarrow \mathbf{v}_t(i) + 1$
else

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    $C_t \leftarrow C_{t-1}$
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end if
if $\mathbf{x}_t$ is closer than R to any $\mathbf{c}_i \in C_t$ **then**
    $\mathbf{v}_t(i) \leftarrow \mathbf{v}_t(i) + 1$
else
    $\mathbf{v}_t(|C_t| + 1) \leftarrow 1$
end if

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Doubling algorithm [**charikar1997incremental**]

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C_t is changing. How far can $\mathbf{x}$ be from some $\mathbf{c}$?

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$$R + \frac{R}{R} + \frac{R}{R^2} + \cdots = R\left(1 + \frac{1}{R} + \frac{1}{R^2} + \cdots\right)$$

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Guarantees

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Guarantees: 8-approximation algorithm.

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Guarantees: 8-approximation algorithm.

Why not incremental k-means?

Online SSL with Graphs

Video examples

http://www.bkveton.com/videos/Coffee.mp4

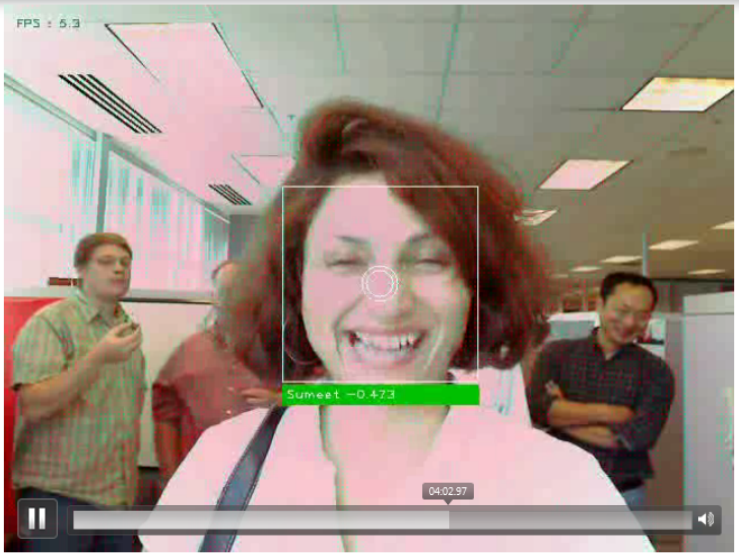
http://www.bkveton.com/videos/Ad.mp4

https://misovalko.github.io/projects/courses/graphsML/lectures/videos/kveton2009nipsdemo.adaptation.mov

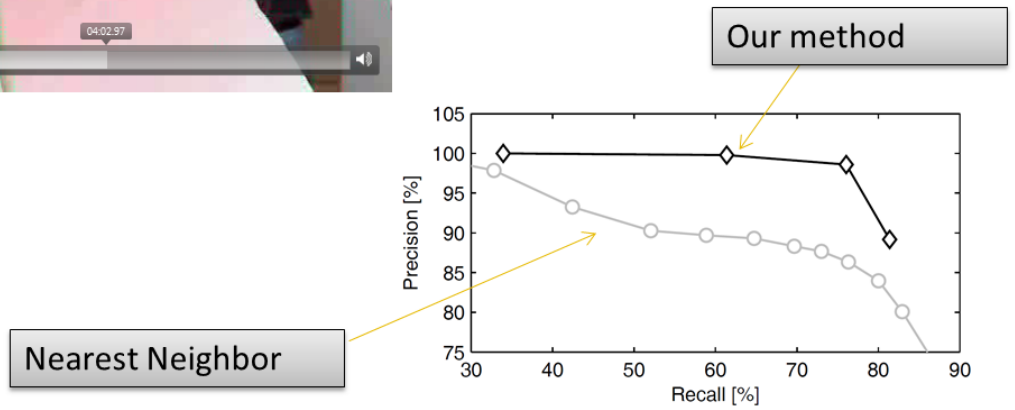
https://misovalko.github.io/projects/courses/graphsML/lectures/videos/kveton2009nipsdemo.officespace.mov

https://misovalko.github.io/projects/courses/graphsML/lectures/videos/press-intel-2015.mp4

SSL with Graphs: Some experimental results



- 8 people classification
- Making funny faces
- 4 faces/person are labeled



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https://misovalko.github.io/mva-ml-graphs.html