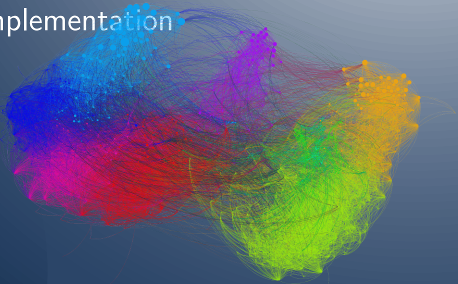
Graphs in Machine Learning

Google PageRank: Computation

Power Method and Large-Scale Implementation

Michal Valko

Inria & ENS Paris-Saclay, MVA



Partially based on material by: Andreas Krause, Branislav Kveton, Michael Kearns

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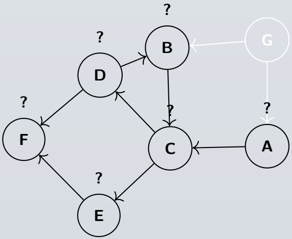

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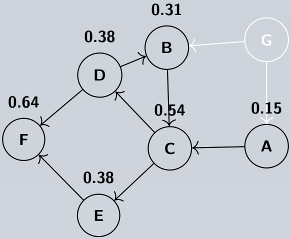

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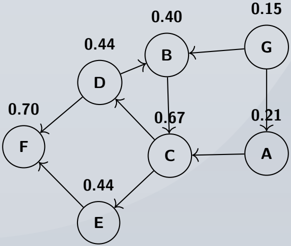

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- All search engines combined process more than 500 million queries per day

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we store only $\mathbf{M}$ but do computations as with $\mathbf{G}$

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https://misovalko.github.io/mva-ml-graphs.html